Lecture 3

2017/2018

Microwave Devices and Circuits for Radiocommunications

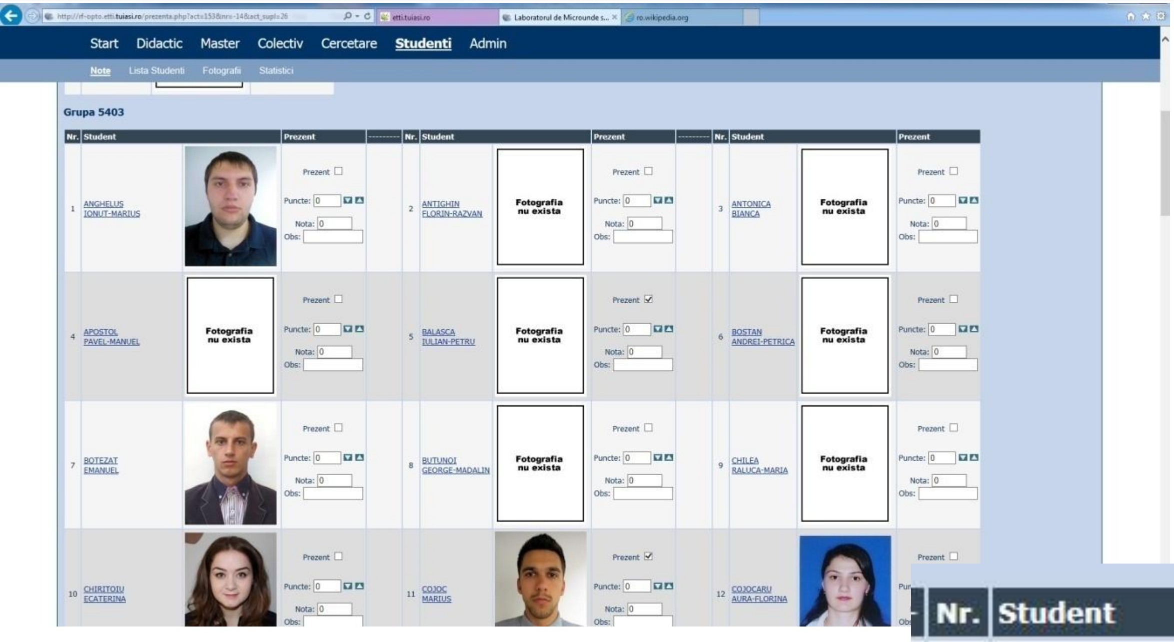
2017/2018

- 2C/1L, **MDCR**
- **Attendance at minimum 7 sessions (course + laboratory)**
- Lectures- **assistant professor Radu Damian**
  - Monday 16-18, P2
  - E – 50% final grade
  - **problems** + (**?** 1 topic teory) + (2p atten. lect.) + (3 tests) + (bonus activity)
    - 3p=+0.5p
  - **all materials/equipments authorized**
- Laboratory – **assistant professor Radu Damian**
  - Monday 18-20 II.12 even weeks
  - Thursday 8-14 odd weeks II.12 **?**
  - L – 25% final grade
  - P – 25% final grade

Materials

- RF-OPTO
  - **http://rf-opto.etti.tuiasi.ro**
- **David Pozar, "Microwave Engineering"**, Wiley; 4th edition , 2011
  - 1 exam problem ← Pozar
- Photos
  - sent by email: rdamian@etti.tuiasi.ro
  - used at lectures/laboratory

Photos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BOSTAN ANDREI-PETRICA
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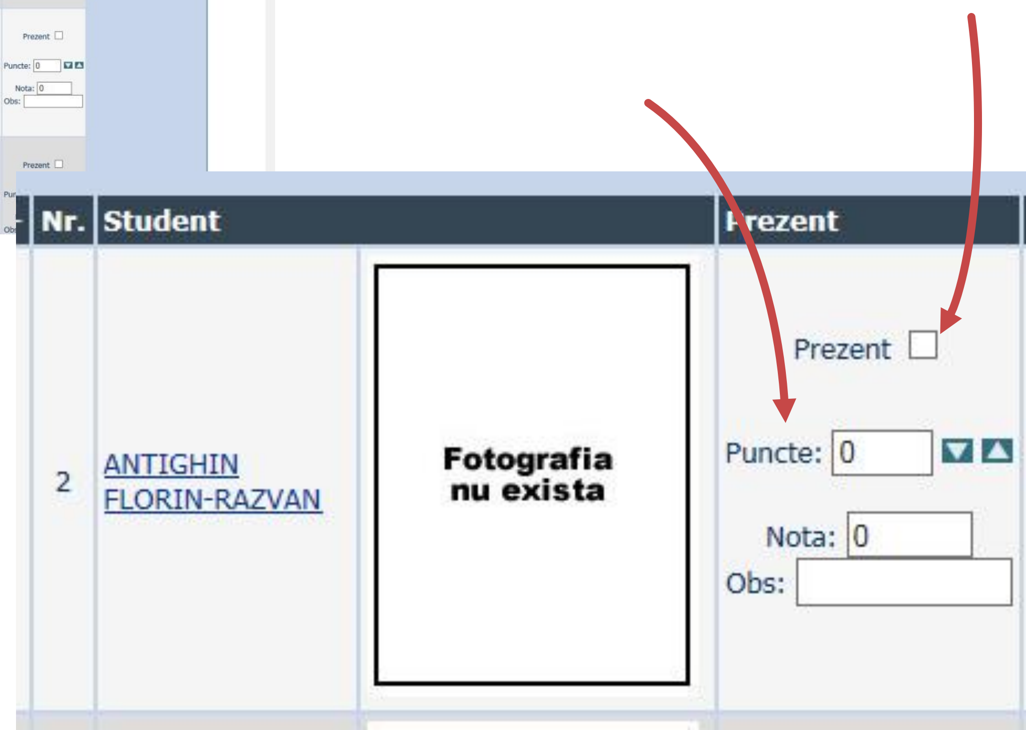
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Access

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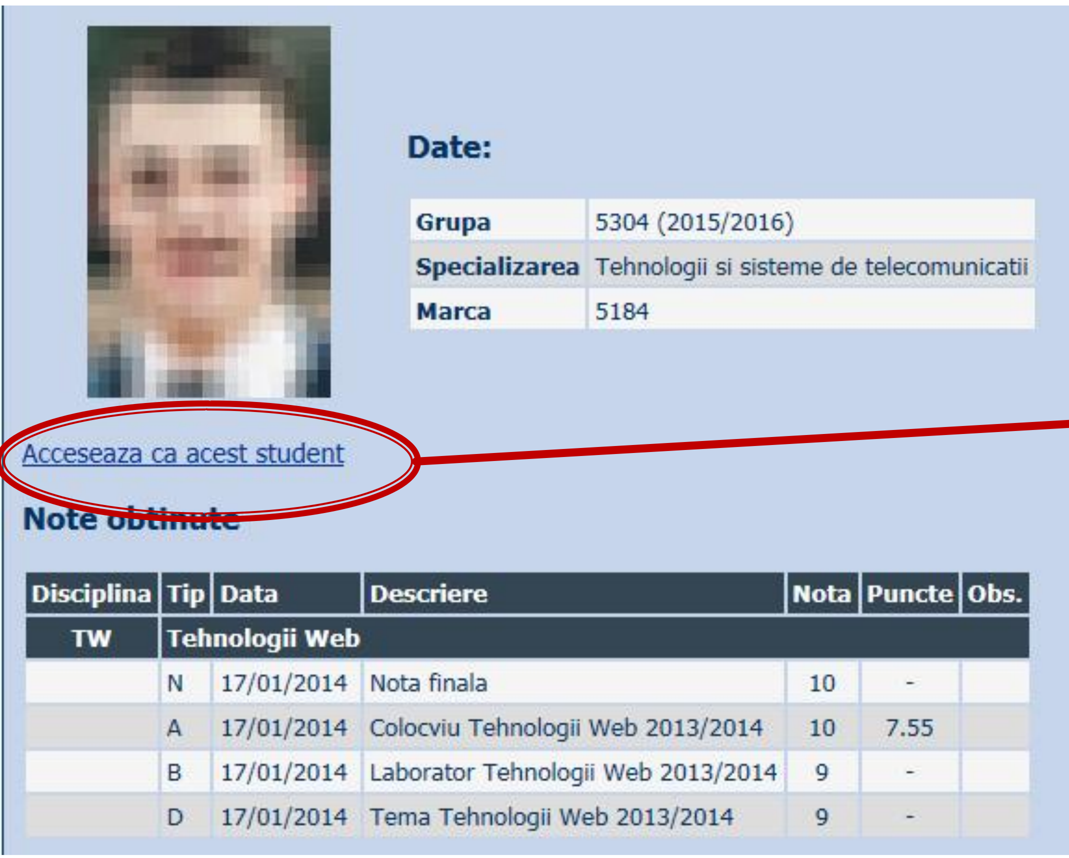


Date:

Grupa	5304 (2015/2016)
Specializarea	Tehnologii si sisteme de telecomunicatii
Marca	5184

Acceseaza ca acest student

Note obtinute

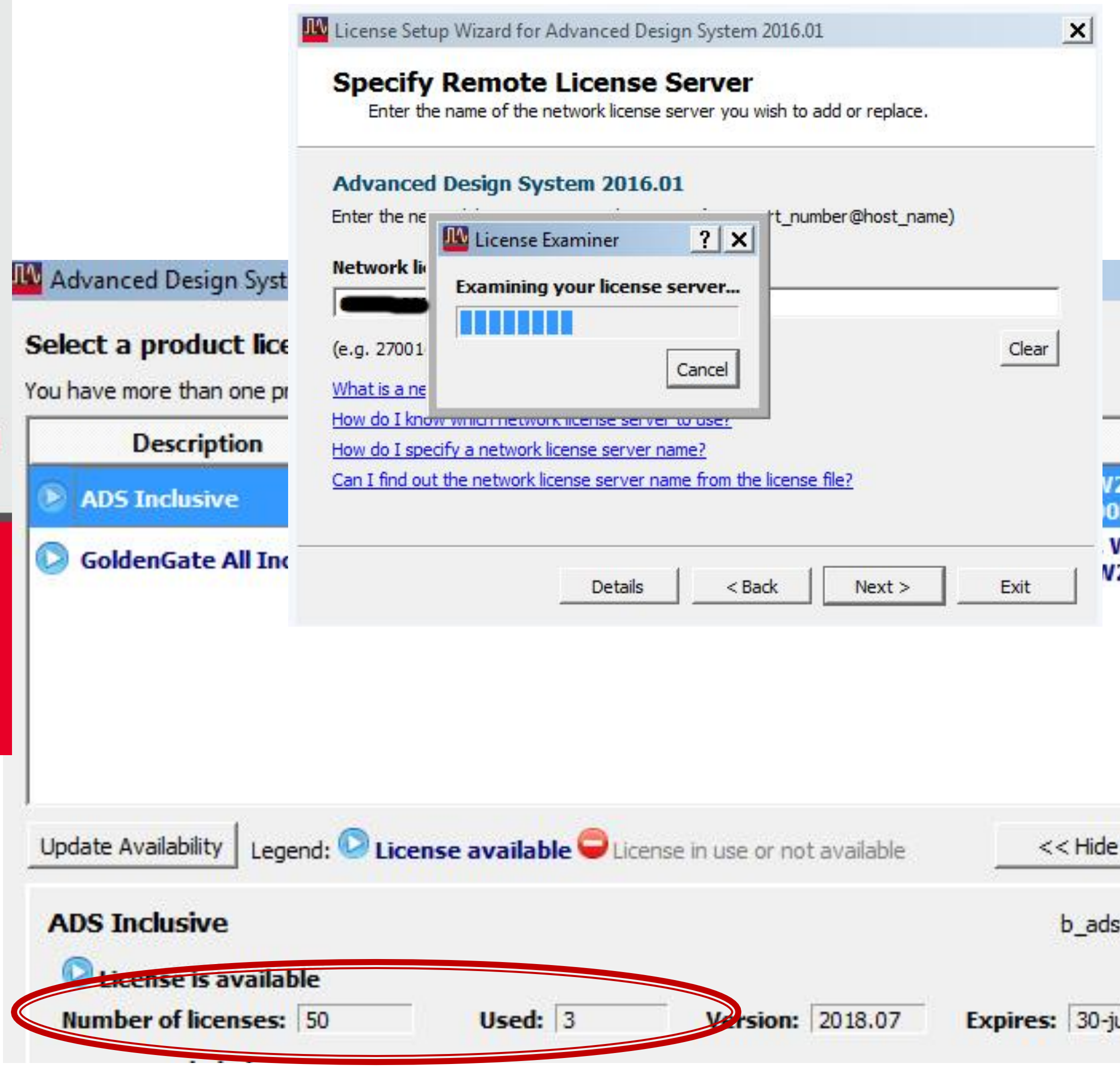
Disciplina	Tip	Data	Descriere	Nota	Puncte	Obs.
TW	Tehnologii Web					
	N	17/01/2014	Nota finala	10	-	
	A	17/01/2014	Colocviu Tehnologii Web 2013/2014	10	7.55	
	B	17/01/2014	Laborator Tehnologii Web 2013/2014	9	-	
	D	17/01/2014	Tema Tehnologii Web 2013/2014	9	-	

Software

- ADS 2016
- EmPro 2015
- based on IP from outside university or campus

Software


Advanced Design System
Premier High-Frequency and High Speed Design Platform
2016.01
KEYSIGHT TECHNOLOGIES
© Keysight Technologies 1985-2016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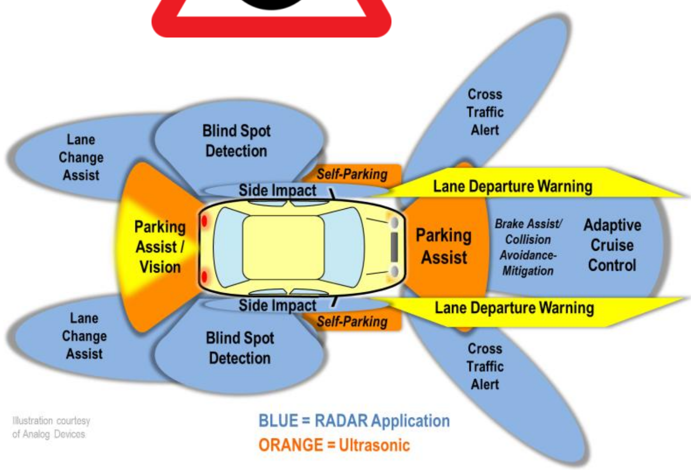
Course Objectives 4

"Engineering"
Sinapses

Technology

> 2010



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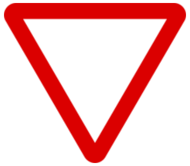


Examen

- Complex numbers arithmetic**!!!!**
- $z = a + j \cdot b$; **$j^2 = -1$**

Polar representation

- standard unit for angles – radians
- microwaves traditional unit for angles – **degrees in decimal form** (55.89°)

$$\varphi = \arg(z) = \begin{cases} \arctan\left(\frac{b}{a}\right), & a > 0 \\ \arctan\left(\frac{b}{a}\right) + \pi, & a < 0, b \geq 0 \\ \arctan\left(\frac{b}{a}\right) - \pi, & a < 0, b < 0 \\ \frac{\pi}{2}, -\frac{\pi}{2}, nedefinit & a = 0 \end{cases}$$

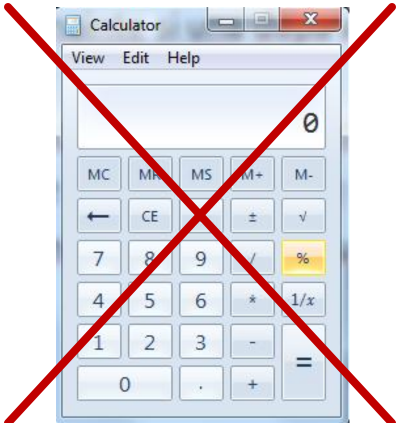
$$\varphi[°] = 180° \cdot \frac{\varphi[rad]}{\pi} \qquad \varphi[rad] = \pi \cdot \frac{\varphi[°]}{180°}$$

Polar representation

- **Attention to angle numerical values!!**
  - math software – work in standard unit: radians
    - a **conversion** is necessary before and after using a trigonometric function (sin, cos, tan, atan, tanh)
  - scientific calculators have the built-in option of choosing the angle unit
    - always **double check** current working unit

$$\varphi[^\circ] = 180^\circ \cdot \frac{\varphi[rad]}{\pi}$$

$$\varphi[rad] = \pi \cdot \frac{\varphi[^\circ]}{180^\circ}$$



Logarithmic scales

$dB = 10 \cdot \log_{10} (P_2 / P_1)$

0 dB	= 1
+ 0.1 dB	= 1.023 (+2.3%)
+ 3 dB	= 2
+ 5 dB	= 3
+ 10 dB	= 10
-3 dB	= 0.5
-10 dB	= 0.1
-20 dB	= 0.01
-30 dB	= 0.001

$dBm = 10 \cdot \log_{10} (P / 1\ mW)$

0 dBm	= 1 mW
3 dBm	= 2 mW
5 dBm	= 3 mW
10 dBm	= 10 mW
20 dBm	= 100 mW
-3 dBm	= 0.5 mW
-10 dBm	= 100 µW
-30 dBm	= 1 µW
-60 dBm	= 1 nW

[dBm] + [dB] = [dBm]

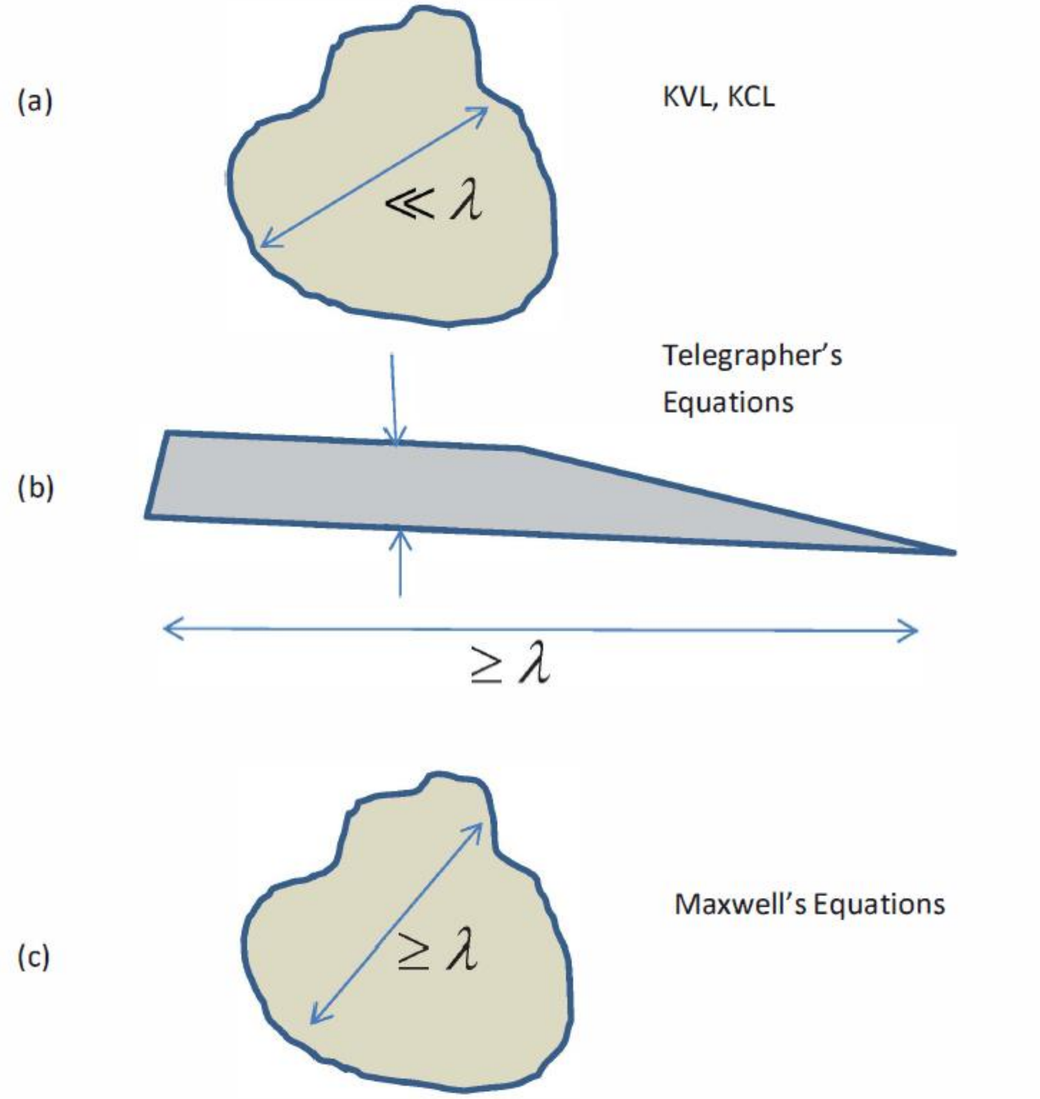
[dBm/Hz] + [dB] = [dBm/Hz]

→ [x] + [dB] = [x]

Electrical Length

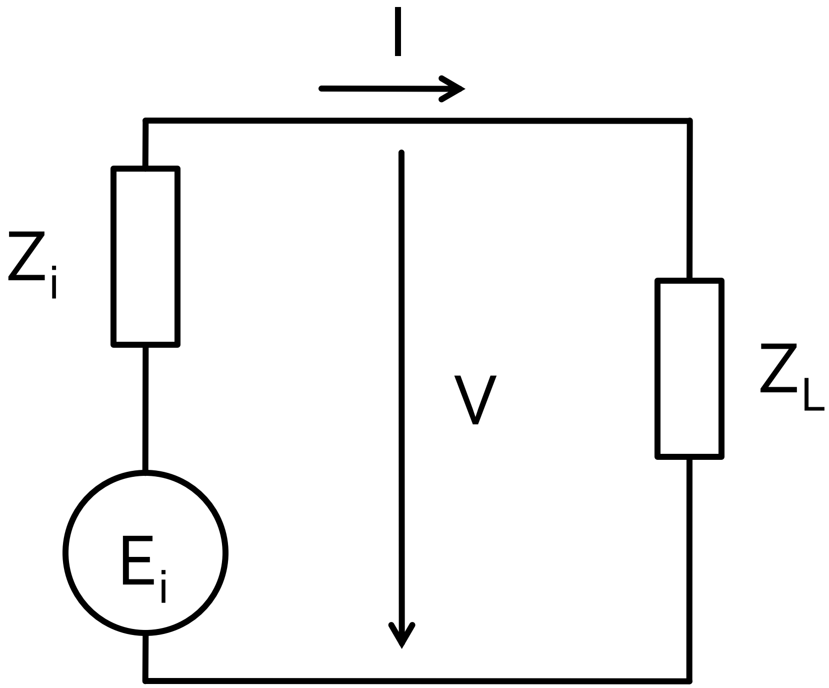
- Behavior (and description) of any circuit depends on his electrical length at the particular frequency of interest
  - E≈0 → Kirchhoff
  - E>0 → wave propagation

$$E = \beta \cdot l = \frac{2\pi}{\lambda} \cdot l = 2\pi \cdot \left(\frac{l}{\lambda} \right)$$



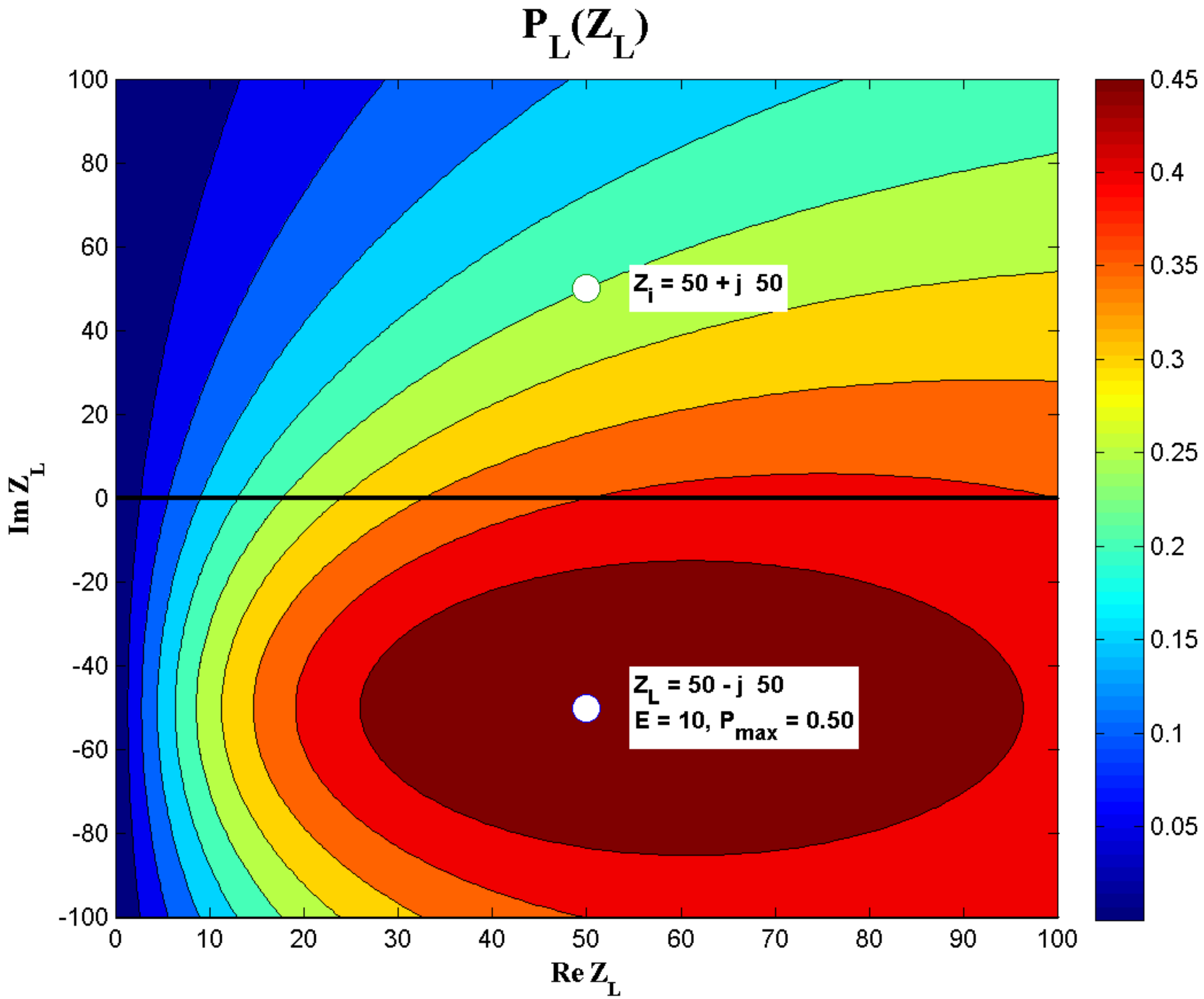
Matching

- Source **matched** to load ?



- impedance values ?
- existence of reflections ?

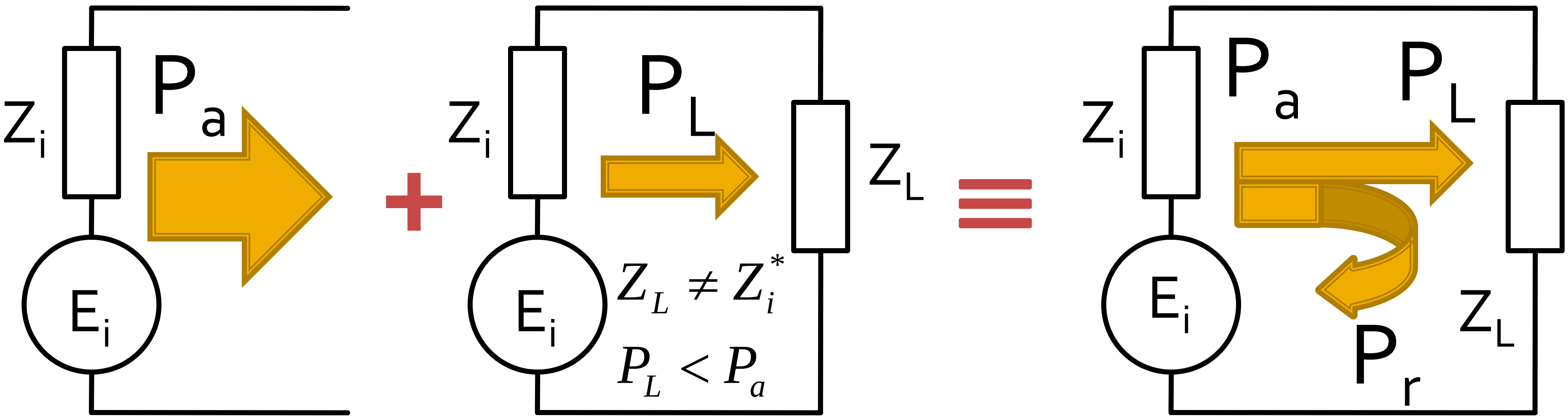
Matching, example



$$Z_L = Z_i^*$$

$$\Gamma_L = \Gamma_i^*$$

Reflection and power / Model

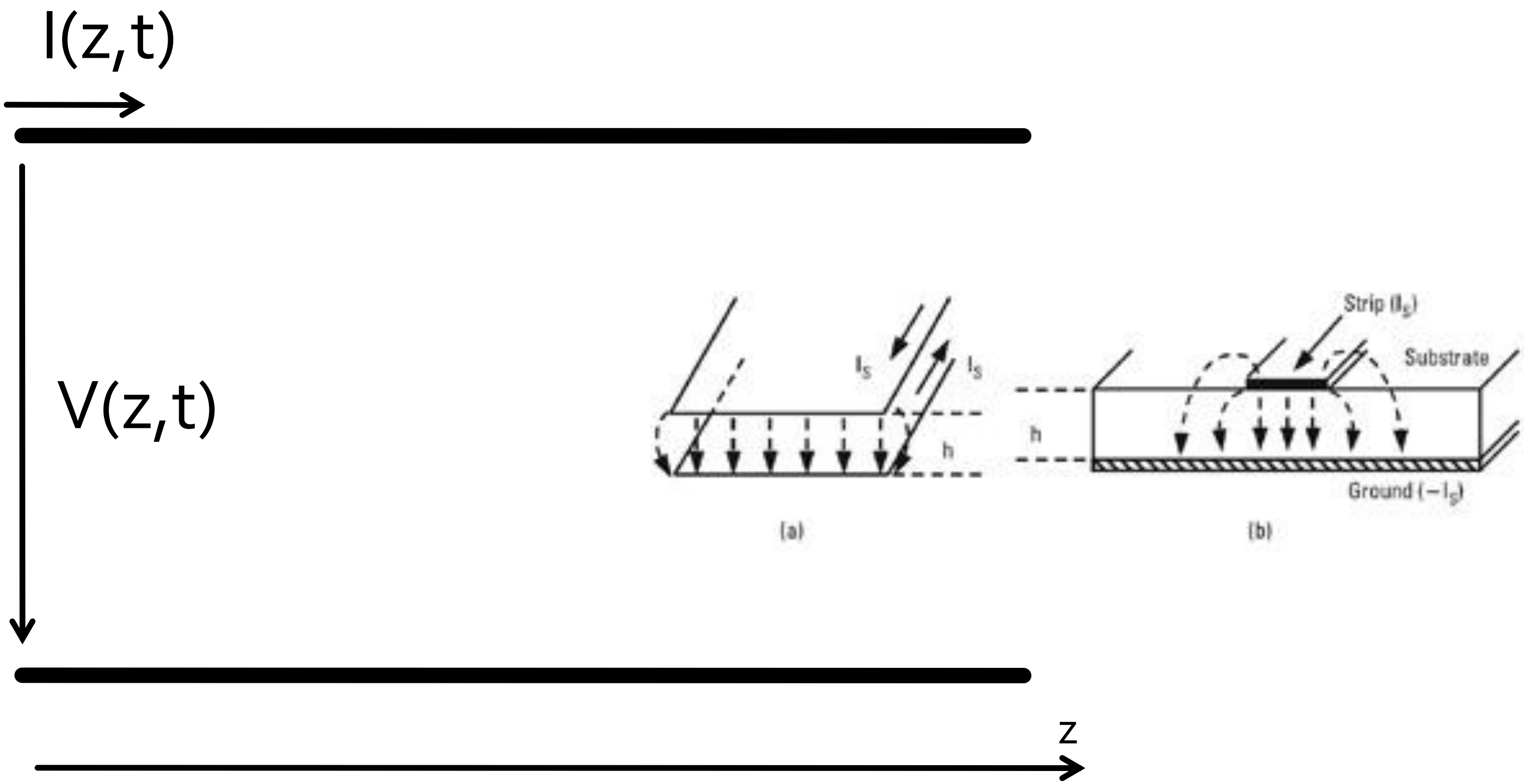


- The source has the ability to sent to the load a certain maximum power (available power) P_a
- For a particular load the power sent to the load is less than the maximum (mismatch) $P_L < P_a$
- The phenomenon is **"as if"** (model) some of the power is reflected $P_r = P_a - P_L$
- The power is a **scalar !**

TEM transmission lines

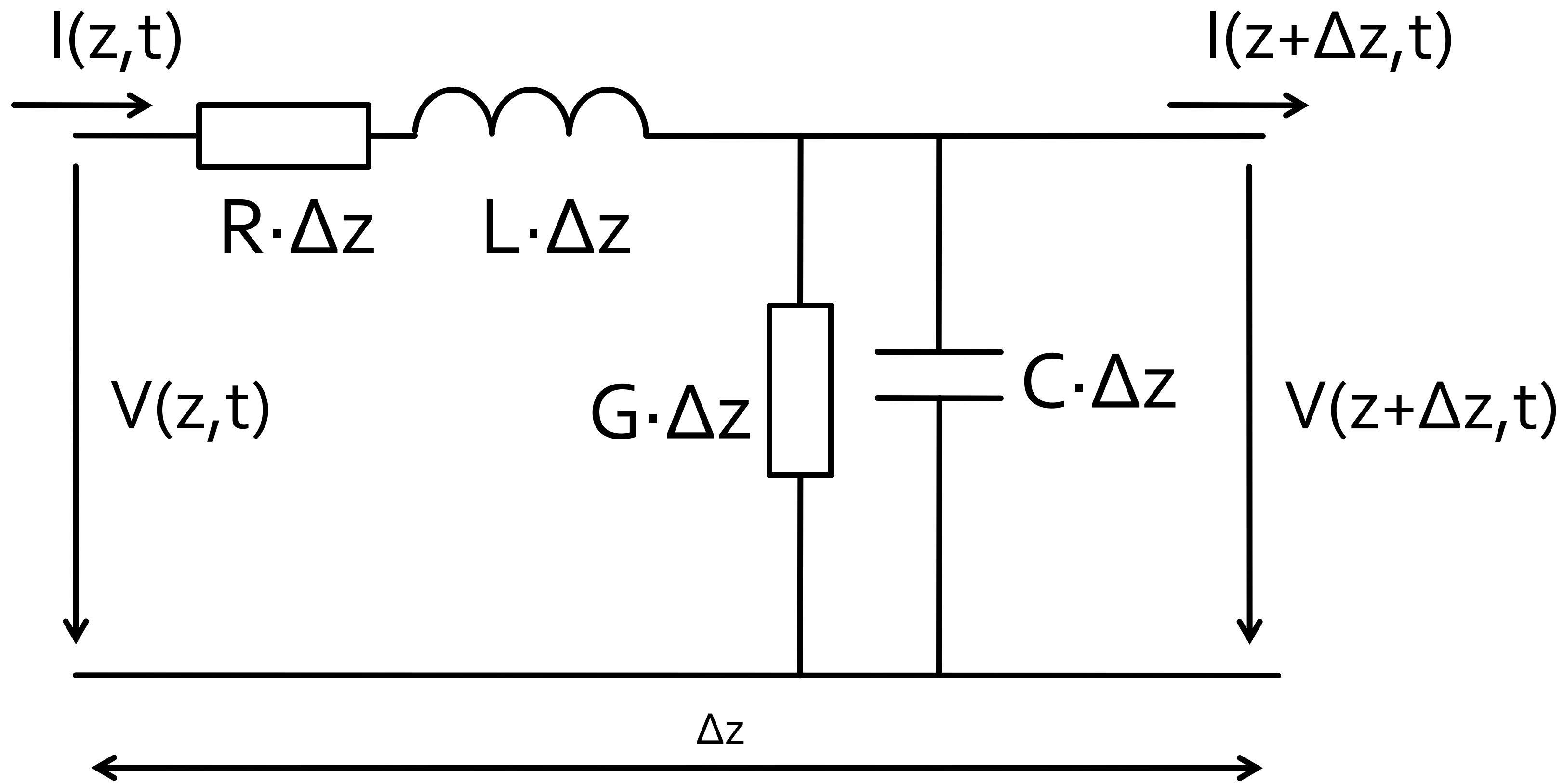
Transmission line

- TEM wave propagation, at least two conductors



Transmission line equivalent model

- TEM wave propagation, at least two conductors



Telegrapher equations

- time domain

$$\frac{\partial v(z,t)}{\partial z} = -R \cdot i(z,t) - L \cdot \frac{\partial i(z,t)}{\partial t}$$

$$\frac{\partial i(z,t)}{\partial z} = -G \cdot v(z,t) - C \cdot \frac{\partial v(z,t)}{\partial t}$$

- harmonic signals

$$\frac{dV(z)}{dz} = -(R + j \cdot \omega \cdot L) \cdot I(z)$$

$$\frac{dI(z)}{dz} = -(G + j \cdot \omega \cdot C) \cdot V(z)$$

Solution

$$\frac{d^2V(z)}{dz^2} - \gamma^2 \cdot V(z) = 0$$

$$\frac{d^2I(z)}{dz^2} - \gamma^2 \cdot I(z) = 0$$

$$\gamma = \alpha + j \cdot \beta = \sqrt{(R + j \cdot \omega \cdot L) \cdot (G + j \cdot \omega \cdot C)}$$

$$\nabla^2 E - \gamma^2 E = 0$$

$$\nabla^2 H - \gamma^2 H = 0$$

$$\gamma^2 = -\omega^2 \varepsilon\mu + j\omega\mu\sigma$$

$$E_y = E_+ e^{-\gamma \cdot z} + E_- e^{\gamma \cdot z}$$

Solutions

$$\begin{cases} V(z) = V_0^+ e^{-\gamma\cdot z} + V_0^- e^{\gamma\cdot z} \\ I(z) = I_0^+ e^{-\gamma\cdot z} + I_0^- e^{\gamma\cdot z} \end{cases}$$

$$\gamma = \alpha + j\cdot\beta = \sqrt{(R + j\cdot\omega\cdot L)\cdot(G + j\cdot\omega\cdot C)}$$

$$V(z) = V_0^+ e^{-\gamma\cdot z} + V_0^- e^{\gamma\cdot z}$$

$$\frac{dV(z)}{dz} = -(R + j\cdot\omega\cdot L)\cdot I(z)$$

$$I(z) = \frac{\gamma}{R + j\cdot\omega\cdot L}\left(V_0^+ e^{-\gamma\cdot z} - V_0^- e^{\gamma\cdot z}\right)$$

$$Z_0 \equiv \frac{R + j\cdot\omega\cdot L}{\gamma} = \sqrt{\frac{R + j\cdot\omega\cdot L}{G + j\cdot\omega\cdot C}}$$

- Characteristic impedance of the line

$$\frac{V_0^+}{I_0^+} = Z_0 = -\frac{V_0^-}{I_0^-}$$

$$\lambda = \frac{2\pi}{\beta} \qquad v_f = \frac{\omega}{\beta} = \lambda\cdot f$$

The lossless line

- R=G=0

$$\gamma = \alpha + j\cdot\beta = \sqrt{(R + j\cdot\omega\cdot L)\cdot(G + j\cdot\omega\cdot C)} = j\cdot\omega\cdot\sqrt{L\cdot C}$$

$$\alpha = 0 \quad ; \quad \beta = \omega\cdot\sqrt{L\cdot C}$$

$$Z_0 = \sqrt{\frac{R + j\cdot\omega\cdot L}{G + j\cdot\omega\cdot C}} = \sqrt{\frac{L}{C}}$$

- Z_o is **real**

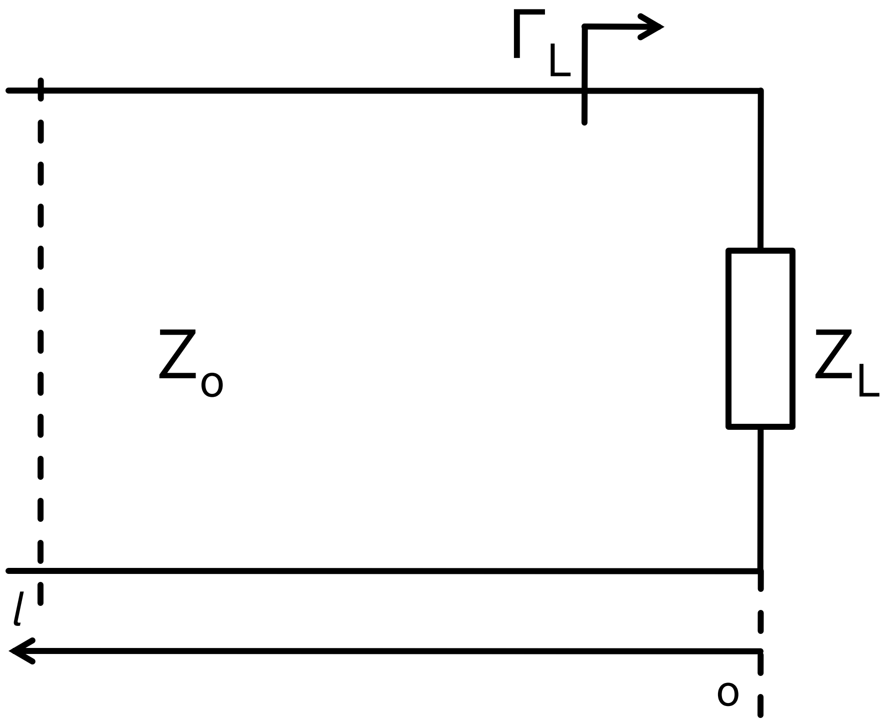
$$V(z) = V_0^+ e^{-j\cdot\beta\cdot z} + V_0^- e^{j\cdot\beta\cdot z}$$

$$I(z) = \frac{V_0^+}{Z_0} e^{-j\cdot\beta\cdot z} - \frac{V_0^-}{Z_0} e^{j\cdot\beta\cdot z}$$

$$\lambda = \frac{2\pi}{\omega\cdot\sqrt{LC}}$$

$$v_f = \frac{1}{\sqrt{LC}}$$

The lossless line



$$V(z) = V_0^+ e^{-j\cdot\beta\cdot z} + V_0^- e^{j\cdot\beta\cdot z}$$

$$I(z) = \frac{V_0^+}{Z_0} e^{-j\cdot\beta\cdot z} - \frac{V_0^-}{Z_0} e^{j\cdot\beta\cdot z}$$

$$Z_L = \frac{V(0)}{I(0)} \qquad Z_L = \frac{V_0^+ + V_0^-}{V_0^+ - V_0^-} \cdot Z_0$$

- voltage reflection coefficient

$$\Gamma = \frac{V_0^-}{V_0^+} = \frac{Z_L - Z_0}{Z_L + Z_0}$$

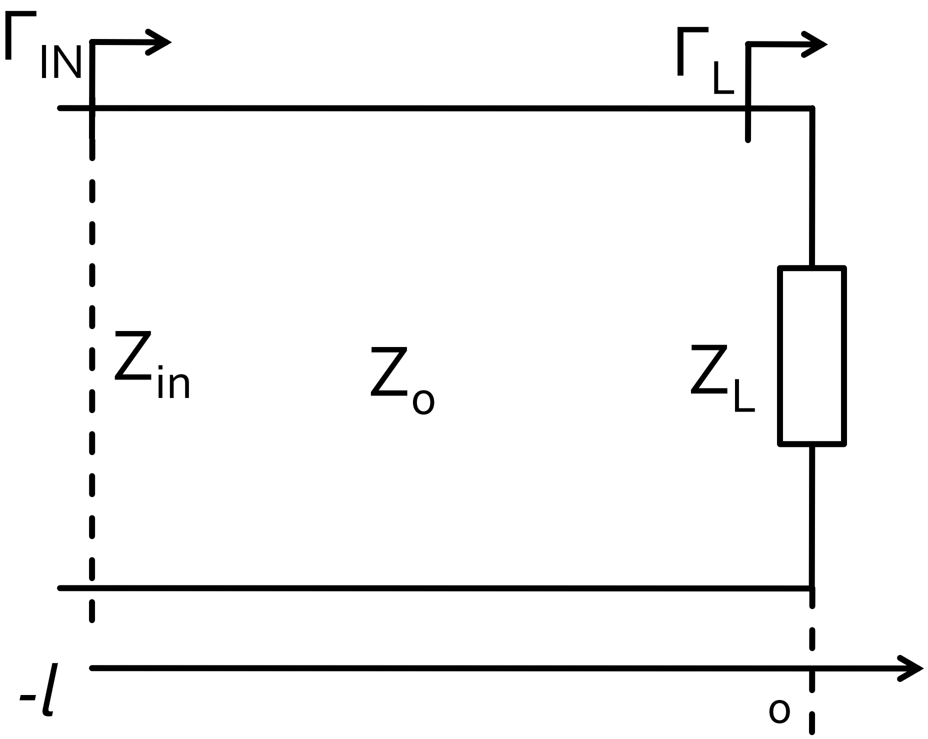
- Z_o real

The lossless line

- voltage reflection coefficient seen at the input of the line

$$V(z) = V_0^+ e^{-j\cdot\beta\cdot z} + V_0^- e^{j\cdot\beta\cdot z}$$

$$\Gamma = \Gamma(z) = \frac{V_0^-(z)}{V_0^+(z)}$$



$$V(0) = V_0^+ + V_0^- \qquad \Gamma(0) = \Gamma_L = \frac{V_0^-}{V_0^+}$$

$$V(-l) = V_0^+ e^{j\cdot\beta\cdot l} + V_0^- e^{-j\cdot\beta\cdot l}$$

$$\Gamma(-l) = \Gamma_{IN} = \frac{V_0^- \cdot e^{-j\cdot\beta\cdot l}}{V_0^+ \cdot e^{j\cdot\beta\cdot l}} = \Gamma(0)\cdot e^{-2j\cdot\beta\cdot l}$$

$$|\Gamma(-l)| = |\Gamma(0)|\cdot\left|e^{-2j\cdot\beta\cdot l}\right| = |\Gamma(0)|$$

$$\boxed{\Gamma(-l) = \Gamma(0)\cdot e^{-2j\cdot\beta\cdot l}}$$

The lossless line

$$V(z) = V_0^+ \cdot \left(e^{-j\cdot\beta\cdot z} + \Gamma \cdot e^{j\cdot\beta\cdot z}\right) \qquad I(z) = \frac{V_0^+}{Z_0} \cdot \left(e^{-j\cdot\beta\cdot z} - \Gamma \cdot e^{j\cdot\beta\cdot z}\right)$$

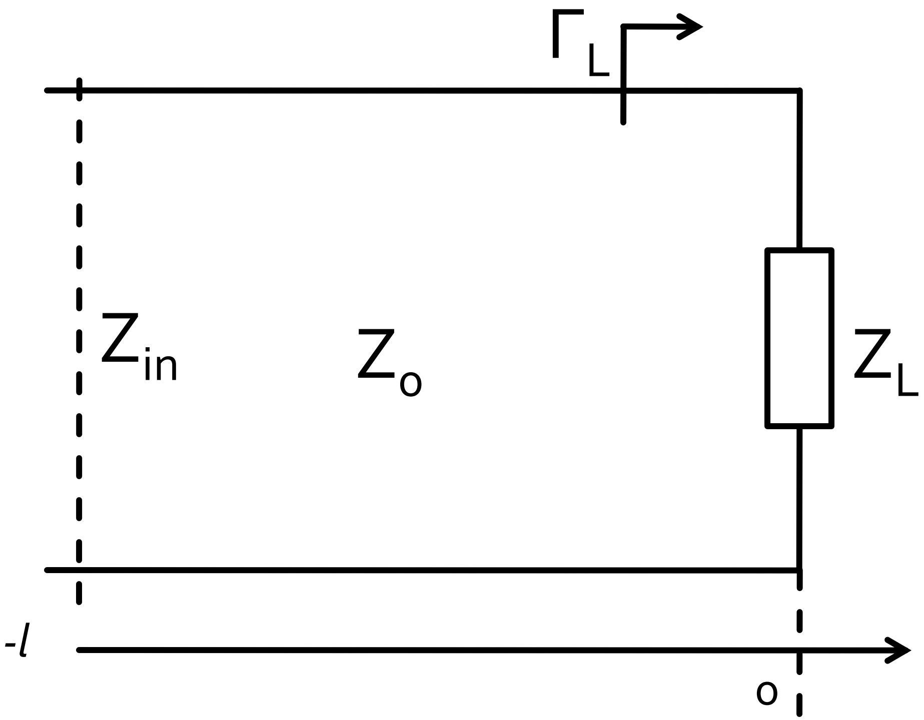
- time-average Power flow along the line

$$P_{\text{avg}} = \frac{1}{2}\text{Re}\{V(z)I(z)^*\} = \frac{1}{2}\frac{|V_o^+|^2}{Z_0}\text{Re}\{1 - \Gamma^* e^{-2j\beta z} + \Gamma e^{2j\beta z} - |\Gamma|^2\}$$

$$P_{\text{avg}} = \frac{1}{2}\frac{|V_o^+|^2}{Z_0}(1 - |\Gamma|^2)$$

- Total power delivered to the load = Incident power – "Reflected" power
- Return "Loss" [dB] $\text{RL} = -20 \log |\Gamma| \text{ dB}$

The lossless line



$$V(-l) = V_0^+ e^{j\cdot\beta\cdot l} + V_0^- e^{-j\cdot\beta\cdot l}$$

$$I(-l) = \frac{V_0^+}{Z_0} e^{j\cdot\beta\cdot l} - \frac{V_0^-}{Z_0} e^{-j\cdot\beta\cdot l}$$

$$Z_{in} = \frac{V(-l)}{I(-l)} \qquad Z_{in} = Z_0 \cdot \frac{1+\Gamma\cdot e^{-2j\cdot\beta\cdot l}}{1-\Gamma\cdot e^{-2j\cdot\beta\cdot l}}$$

- the input impedance seen looking toward the load

$$Z_{in} = Z_0 \cdot \frac{(Z_L + Z_0)\cdot e^{j\cdot\beta\cdot l} + (Z_L - Z_0)\cdot e^{-j\cdot\beta\cdot l}}{(Z_L + Z_0)\cdot e^{j\cdot\beta\cdot l} - (Z_L - Z_0)\cdot e^{-j\cdot\beta\cdot l}}$$

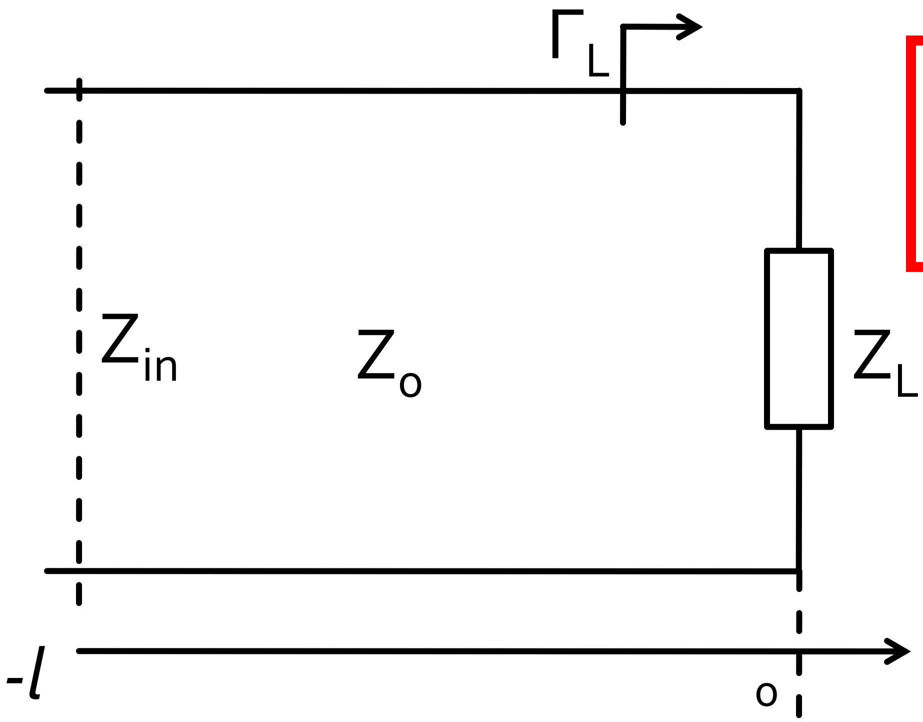
$$Z_{in} = Z_0 \cdot \frac{Z_L + j\cdot Z_0\cdot\tan(\beta\cdot l)}{Z_0 + j\cdot Z_L\cdot\tan(\beta\cdot l)}$$

The lossless line

$$Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan \beta \cdot l}{Z_0 + j \cdot Z_L \cdot \tan \beta \cdot l}$$

The lossless line

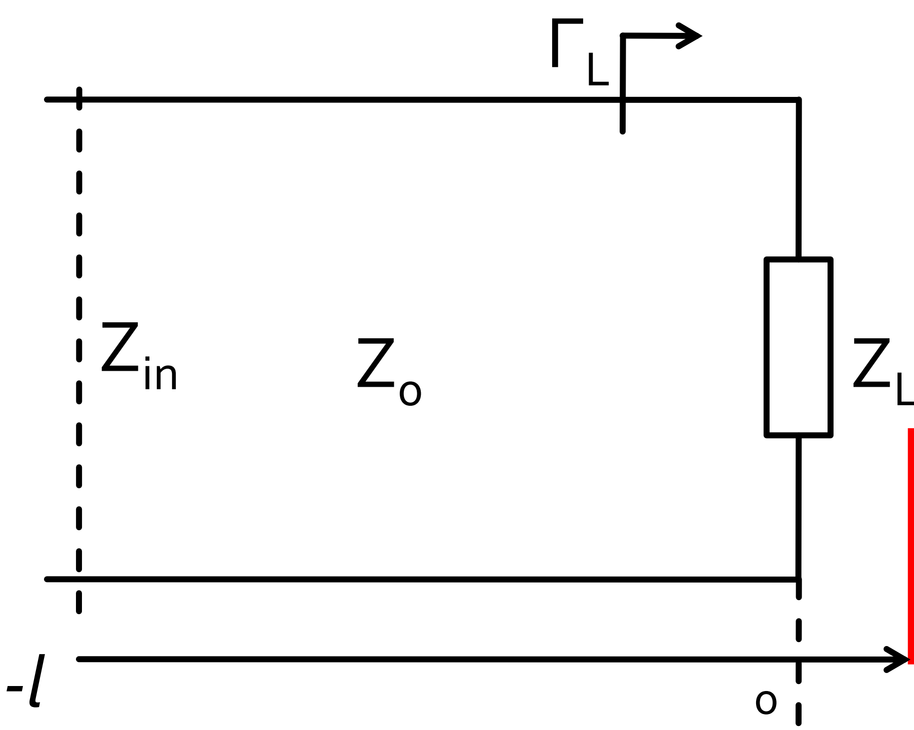
- input impedance of a length ***l*** of transmission line with characteristic impedance ***Z_o***, loaded with an arbitrary impedance ***Z_L***



$$Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan \beta \cdot l}{Z_0 + j \cdot Z_L \cdot \tan \beta \cdot l}$$

The lossless line

- input impedance is **frequency dependent** through $\beta \cdot l$



$$v_f = \frac{\omega}{\beta} = \lambda \cdot f \qquad \lambda = \frac{2\pi}{\beta}$$

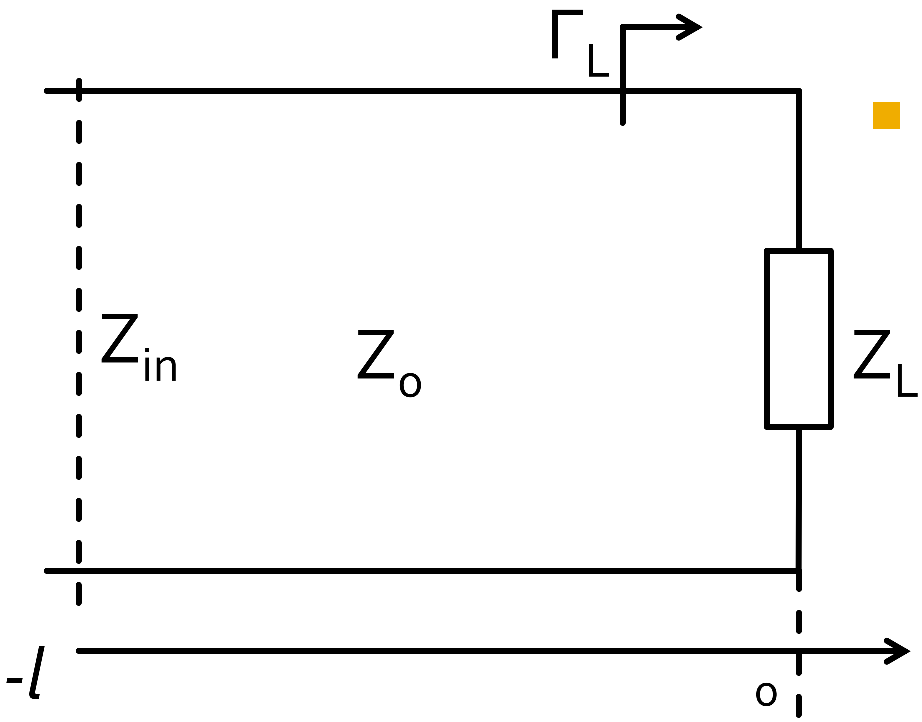
$$\beta \cdot l = \frac{2\pi}{\lambda} \cdot l = \frac{2\pi \cdot f}{v_f} \cdot l = \frac{2\pi \cdot l}{v_f} \cdot f$$

frequency dependence is periodical, imposed by the tan trigonometric function

$$Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan \beta \cdot l}{Z_0 + j \cdot Z_L \cdot \tan \beta \cdot l}$$

The lossless line, special cases

- $l = k \cdot \lambda/2$ $\quad \beta \cdot l = \frac{2\pi}{\lambda} \cdot l = k \cdot \pi \quad \tan \beta \cdot l = 0$ $\quad \boxed{Z_{in} = Z_0}$
- $l = \lambda/4 + k \cdot \lambda/2$ $\quad \tan \beta \cdot l \to \infty$ $\quad \boxed{Z_{in} = \frac{Z_0^2}{Z_L}}$



- quarter-wave transformer

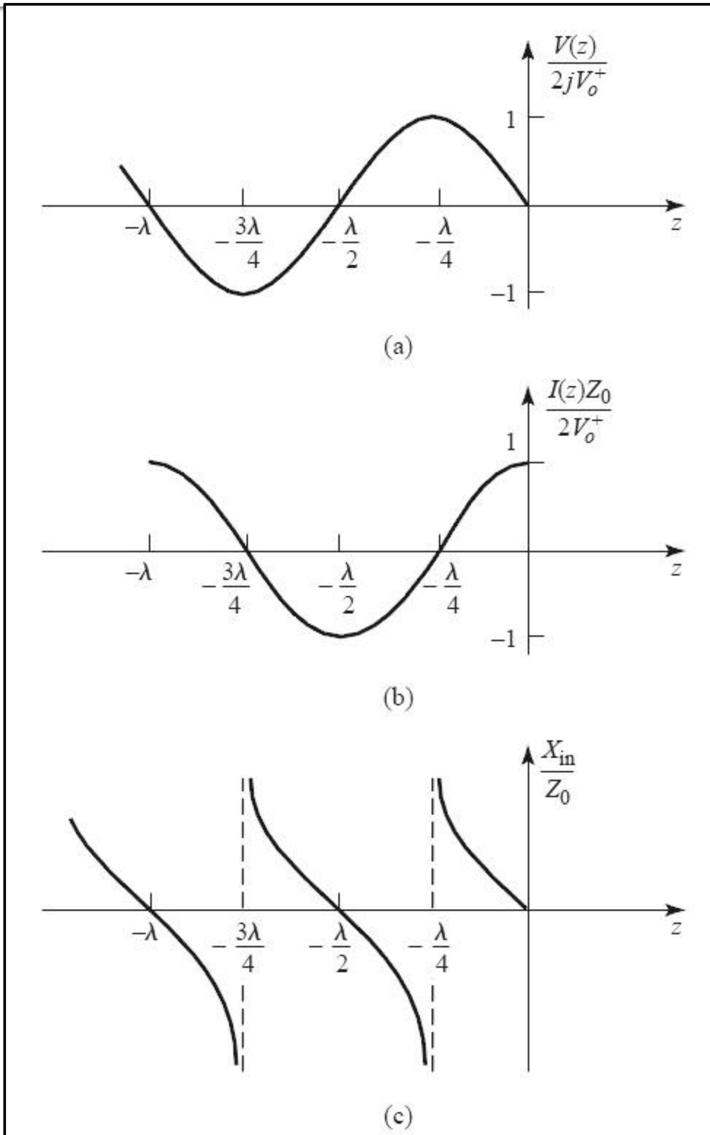
$$\boxed{Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan \beta \cdot l}{Z_0 + j \cdot Z_L \cdot \tan \beta \cdot l}}$$

Short-circuited transmission line

- purely imaginary for any length *l*
  - +/- → depending on *l* value

$$Z_{in} = j \cdot Z_0 \cdot \tan \beta \cdot l$$

$$Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan(\beta \cdot l)}{Z_0 + j \cdot Z_L \cdot \tan(\beta \cdot l)}$$

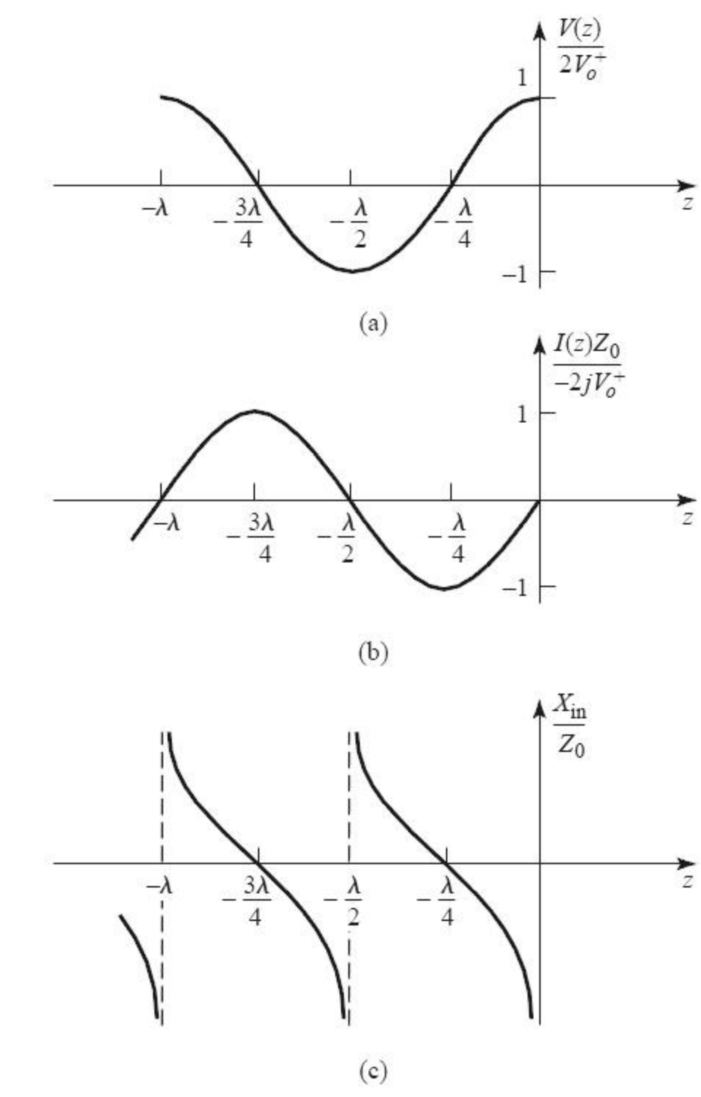


Open-circuited transmission line

- purely imaginary for any length *l*
  - +/- → depending on *l* value

$$Z_{in} = -j \cdot Z_0 \cdot \cot \beta \cdot l$$

$$Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan(\beta \cdot l)}{Z_0 + j \cdot Z_L \cdot \tan(\beta \cdot l)}$$



Voltage standing wave ratio

$$V(z) = V_0^+ \cdot \left(e^{-j\cdot\beta\cdot z} + \Gamma \cdot e^{j\cdot\beta\cdot z}\right) \qquad |V(z)| = |V_0^+| \cdot |e^{-j\cdot\beta\cdot z}| \cdot |1 + \Gamma \cdot e^{2j\cdot\beta\cdot z}| \qquad \Gamma = |\Gamma| \cdot e^{j\theta}$$

$$|V(z)| = |V_0^+| \cdot |1 + |\Gamma| \cdot e^{\theta + 2j\cdot\beta\cdot z}|$$

maximum magnitude value for $e^{\theta + 2j\cdot\beta\cdot z} = 1$ $\qquad V_{max} = |V_0^+| \cdot (1 + |\Gamma|)$

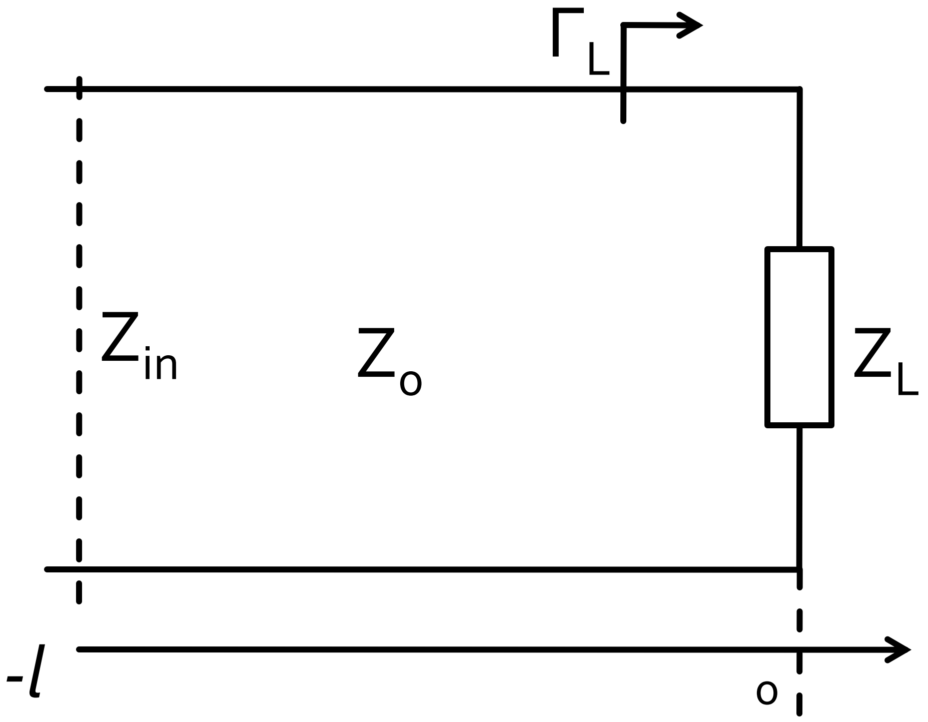
minimum magnitude value for $e^{\theta + 2j\cdot\beta\cdot z} = -1$ $\qquad V_{min} = |V_0^+| \cdot (1 - |\Gamma|)$

- SWR is defined as the ratio between maximum and minimum
  - (Voltage) Standing Wave Ratio

$$VSWR = \frac{V_{max}}{V_{min}} = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

  - real number $1 \leq VSWR < \infty$
  - a measure of the mismatch (SWR = 1 means a matched line)

The lossless line +/-



$$Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan \beta \cdot l}{Z_0 + j \cdot Z_L \cdot \tan \beta \cdot l}$$

$$V(z) = V_0^+ e^{-\gamma \cdot z} + V_0^- e^{\gamma \cdot z}$$

$$I(z) = I_0^+ e^{-\gamma \cdot z} + I_0^- e^{\gamma \cdot z}$$

$$\Gamma(-l) = \Gamma(0) \cdot e^{-2 j \cdot \beta \cdot l}$$

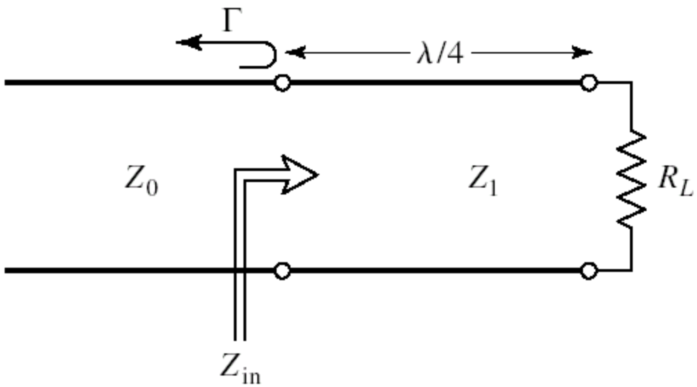
Impedance Matching with Impedance Transformers (Lab 1)

Impedance Matching

The quarter-wave transformer

- Feed line – input line with characteristic impedance Z_o
- Real load impedance R_L
- We desire matching the load to the fider with a second line with the length λ/4 and characteristic impedance Z_1

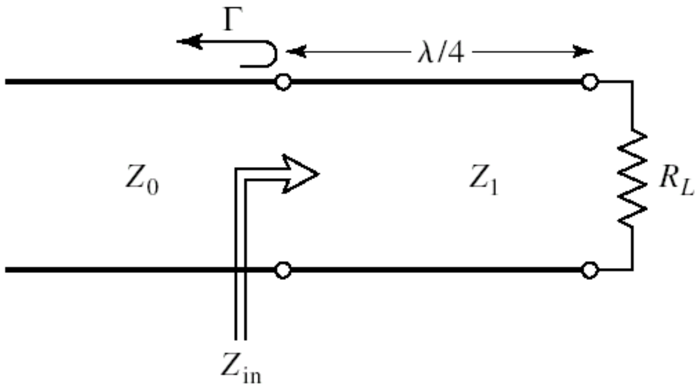
$$Z_{in} = Z_1 \frac{1+\Gamma e^{-2j\beta l}}{1-\Gamma e^{-2j\beta l}}$$



$$\Gamma_O = \frac{V_0^-}{V_0^+} = \frac{R_L - Z_1}{R_L + Z_1}$$

$$Z_{in} = Z_1 \frac{R_L + jZ_1 \tan(\beta l)}{Z_1 + jR_L \tan(\beta l)}$$

The quarter-wave transformer



$$\Gamma_{in} = \frac{Z_{in} - Z_0}{Z_{in} + Z_0}$$

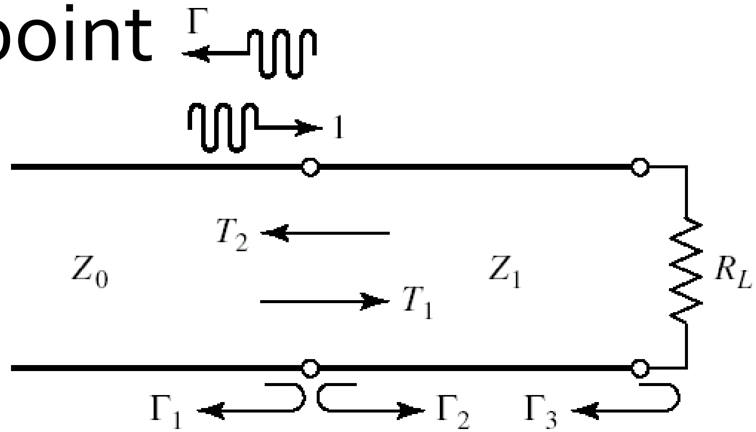
$$\beta \cdot l = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{4} = \frac{\pi}{2} \qquad Z_{in} = \frac{Z_1^2}{R_L}$$

$$\Gamma_{in} = \frac{Z_1^2 - Z_0 \cdot R_L}{Z_1^2 + Z_0 \cdot R_L} \qquad \Gamma_{in} = 0 \quad Z_1 = \sqrt{Z_0 R_L}$$

- In the feed line (Z_0) we have only progressive wave
- In the quarter-wave line (Z_1) we have standing waves

The quarter-wave transformer

- The Multiple-Reflection Viewpoint



$$\Gamma = \Gamma_1 - T_1 T_2 \Gamma_3 + T_1 T_2 \Gamma_2 \Gamma_3^2 - T_1 T_2 \Gamma_2^2 \Gamma_3^3 + \cdots$$

$$= \Gamma_1 - T_1 T_2 \Gamma_3 \sum_{n=0}^{\infty} (-\Gamma_2 \Gamma_3)^n.$$

$$\Gamma_1 = \frac{Z_1 - Z_0}{Z_1 + Z_0},$$

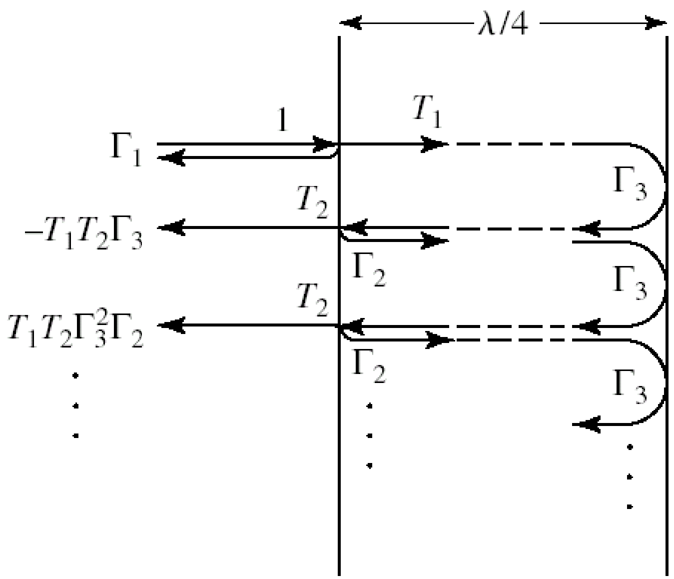
$$\Gamma_2 = \frac{Z_0 - Z_1}{Z_0 + Z_1} = -\Gamma_1,$$

$$\Gamma_3 = \frac{R_L - Z_1}{R_L + Z_1},$$

$$T_1 = \frac{2Z_1}{Z_1 + Z_0},$$

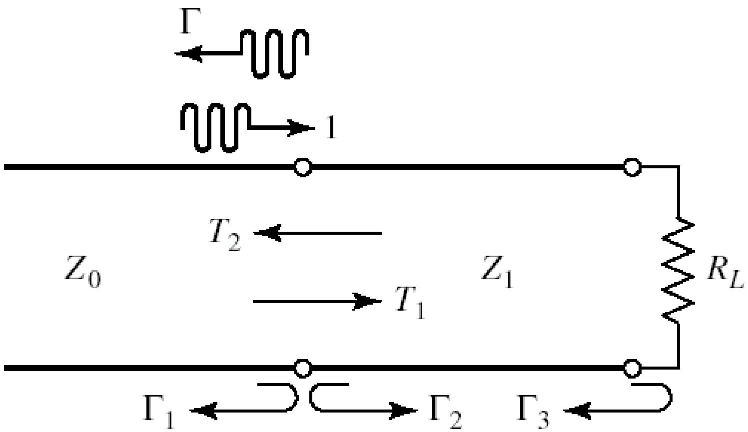
$$T_2 = \frac{2Z_0}{Z_1 + Z_0}.$$

$$T = 1 - \Gamma$$



The quarter-wave transformer

- The Multiple-Reflection Viewpoint

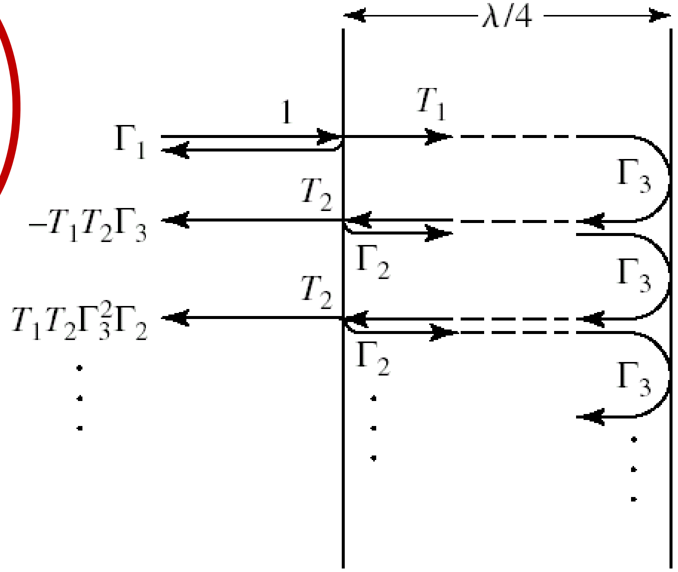


$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}, \qquad \text{for } |x| < 1,$$

$$\Gamma = \Gamma_1 - \frac{T_1 T_2 \Gamma_3}{1+\Gamma_2\Gamma_3} = \frac{\Gamma_1 + \Gamma_1\Gamma_2\Gamma_3 - T_1 T_2 \Gamma_3}{1+\Gamma_2\Gamma_3}.$$

$$\Gamma_1 - \Gamma_3\left(\Gamma_1^2 + T_1 T_2\right) = \frac{2\left(Z_1^2 - Z_0 R_L\right)}{(Z_1+Z_0)(R_L+Z_1)},$$

$$\Gamma = 0 \leftrightarrow Z_1^2 - Z_0 \cdot R_L = 0$$



Frequency response

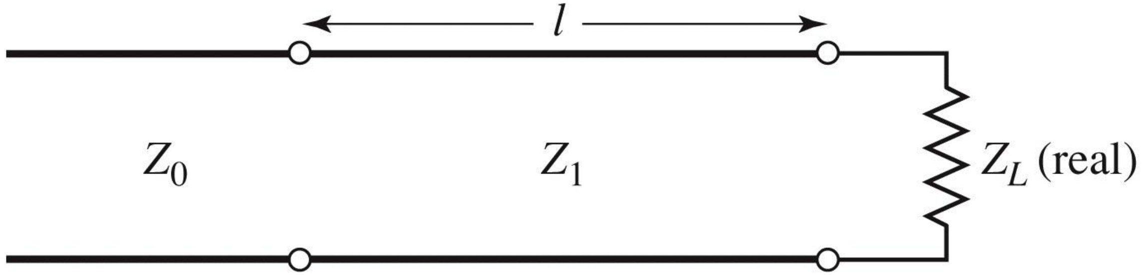


Figure 5.10

$$Z_1 = \sqrt{Z_0 \cdot Z_L}$$

- (**only**) at f_o

$$l = \frac{\lambda_0}{4} \qquad \beta_0 \cdot l = \frac{2\pi}{\lambda_0} \cdot \frac{\lambda_0}{4} = \frac{\pi}{2}$$

$$Z_{in} = Z_1 \cdot \frac{Z_L + j \cdot Z_1 \cdot \tan(\beta \cdot l)}{Z_1 + j \cdot Z_L \cdot \tan(\beta \cdot l)} \qquad \theta \overset{not}{=} \beta \cdot l \qquad t \overset{not}{=} \tan(\beta \cdot l)$$

$$Z_{in} = Z_1 \cdot \frac{Z_L + j \cdot Z_1 \cdot t}{Z_1 + j \cdot Z_L \cdot t}$$

$$\Gamma = \frac{Z_{\text{in}} - Z_0}{Z_{\text{in}} + Z_0} = \frac{Z_1(Z_L - Z_0) + jt\left(Z_1^2 - Z_0 Z_L\right)}{Z_1(Z_L + Z_0) + jt\left(Z_1^2 + Z_0 Z_L\right)}.$$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0 + j2t\sqrt{Z_0 Z_L}}.$$

Frequency response

- matching quality ≡ power reflection coefficient

$$
\begin{aligned}
|\Gamma| &= \frac{|Z_L - Z_0|}{\left[(Z_L + Z_0)^2 + 4t^2 Z_0 Z_L\right]^{1/2}} \\
&= \frac{1}{\left\{(Z_L + Z_0)^2/(Z_L - Z_0)^2 + [4t^2 Z_0 Z_L/(Z_L - Z_0)^2]\right\}^{1/2}} \\
&= \frac{1}{\left\{1 + [4Z_0 Z_L/(Z_L - Z_0)^2] + [4Z_0 Z_L t^2/(Z_L - Z_0)^2]\right\}^{1/2}} \\
&= \frac{1}{\left\{1 + [4Z_0 Z_L/(Z_L - Z_0)^2]\ \sec^2\theta\right\}^{1/2}},
\end{aligned}
$$

$$\sec\theta = \frac{1}{\cos\theta} \rightarrow$$

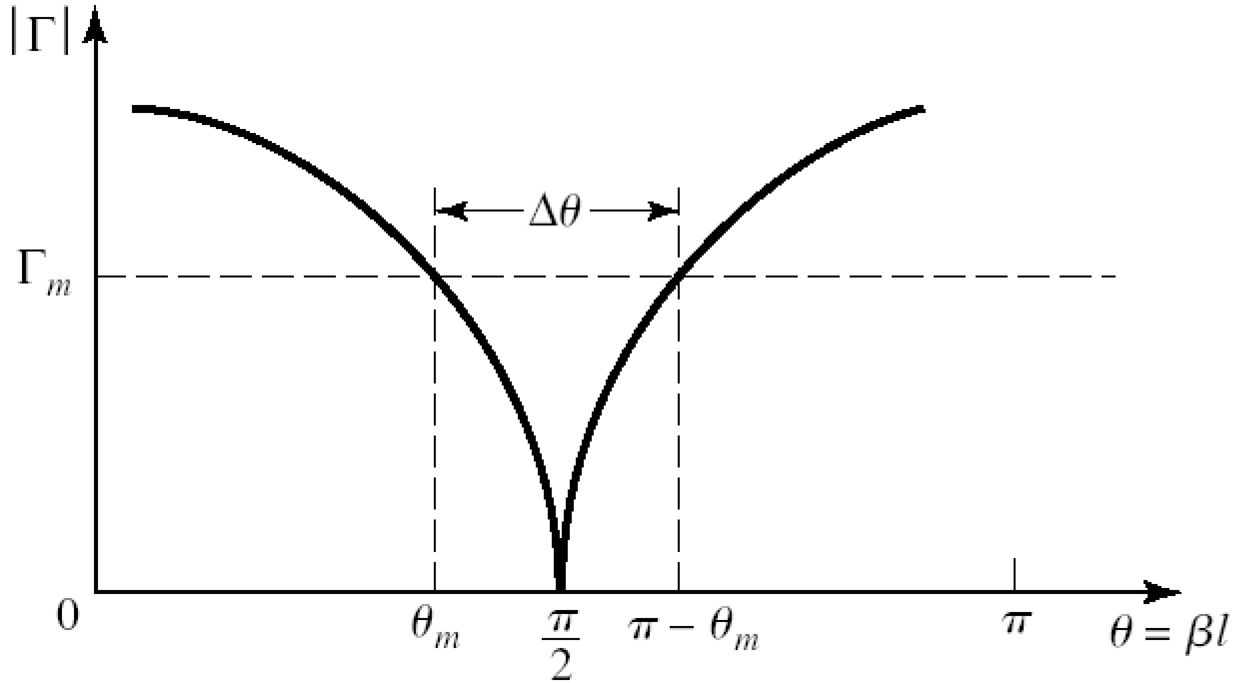
$$\sec^2\theta = 1 + \tan^2\theta = 1 + t^2$$

Frequency response

- we assume that the operating frequency is near the design frequency (narrow bandwidth)

$$f \approx f_0 \qquad l \approx \frac{\lambda_0}{4} \qquad \theta \approx \frac{\pi}{2} \qquad \sec^2\theta = 1 + \tan^2\theta >> 1$$

$$|\Gamma| \cong \frac{|Z_L - Z_0|}{2 \cdot \sqrt{Z_0 \cdot Z_L}} \cdot |\cos\theta|$$



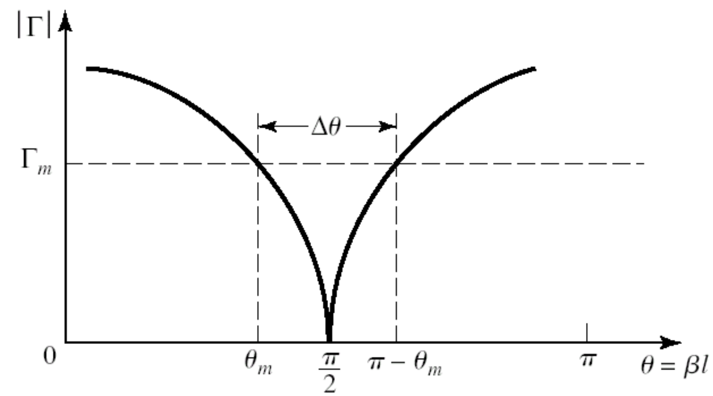
Frequency response

- we set a maximum value Γ_m for an acceptable reflection coefficient magnitude then the bandwidth of the matching transformer, θ_m

$$\frac{1}{\Gamma_m^2} = 1 + \left(\frac{2\sqrt{Z_0 Z_L}}{Z_L - Z_0}\sec\theta_m\right)^2,$$

$$\cos\theta_m = \frac{\Gamma_m}{\sqrt{1-\Gamma_m^2}}\frac{2\sqrt{Z_0 Z_L}}{|Z_L - Z_0|}.$$

$$\Delta\theta = 2\left(\frac{\pi}{2} - \theta_m\right)$$



- for TEM lines

$$\theta = \beta\cdot l = \beta\cdot\frac{\lambda_0}{4} = \frac{2\pi\cdot f}{v_f}\cdot\frac{1}{4}\cdot\frac{v_f}{f_0} = \frac{\pi\cdot f}{2f_0}$$

$$f_m = \frac{2\cdot\theta_m\cdot f_0}{\pi}$$

$$\frac{\Delta f}{f_0} = \frac{2\cdot(f_0 - f_m)}{f_0} = 2 - \frac{4\cdot\theta_m}{\pi} = 2 - \frac{4}{\pi}\cdot\cos^{-1}\left[\frac{\Gamma_m}{\sqrt{1-\Gamma_m^2}}\cdot\frac{2\sqrt{Z_0\cdot Z_L}}{|Z_L - Z_0|}\right]$$

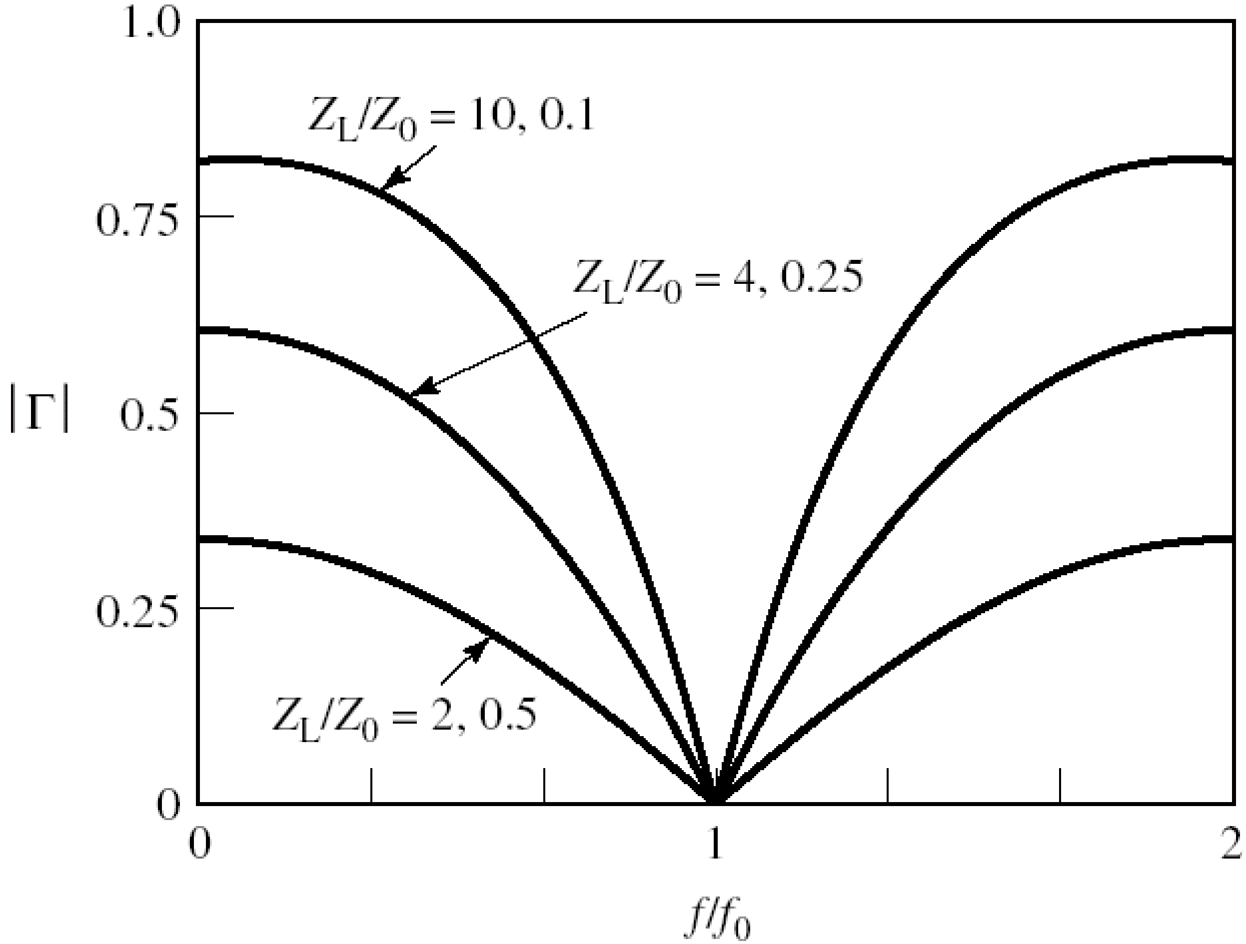
Frequency response

- When non-TEM lines (such as waveguides) are used, the propagation constant is no longer a linear function of frequency, and the wave impedance will be frequency dependent, but in practice the bandwidth of the transformer is often small enough that these complications do not substantially affect the result
- We ignored also the effect of reactances associated with discontinuities when there is a step change in the dimensions of a transmission line ($Z_o \rightarrow Z_1$). This can often be compensated by making a small adjustment in the length of the matching section

Frequency response

- Bandwidth depends on the initial mismatch

increased bandwidth for smaller load mismatches



Exemple

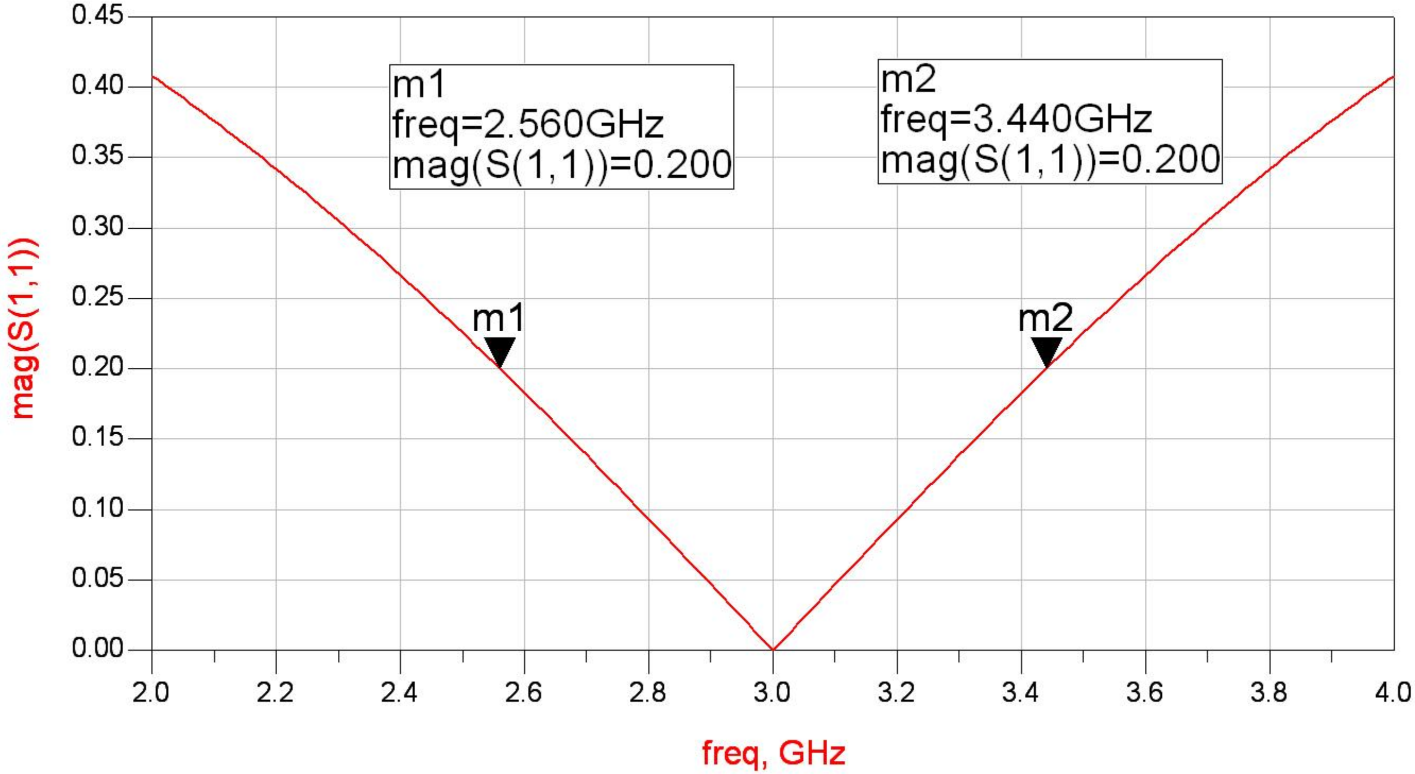
- A quarter-wave matching transformer to match a 10Ω load to a 50 Ω transmission line at f_0=3GHz
  - Determine the percent bandwidth for SWR<1.5

$$Z_1 = \sqrt{Z_0 Z_L} = \sqrt{(50)(10)} = 22.36\ \Omega, \qquad \Gamma_m = \frac{\text{SWR} - 1}{\text{SWR} + 1} = \frac{1.5 - 1}{1.5 + 1} = 0.2.$$

$$\begin{aligned}
\frac{\Delta f}{f_0} &= 2 - \frac{4}{\pi} \cos^{-1}\left[\frac{\Gamma_m}{\sqrt{1 - \Gamma_m^2}} \frac{2\sqrt{Z_0 Z_L}}{|Z_L - Z_0|}\right] \\
&= 2 - \frac{4}{\pi} \cos^{-1}\left[\frac{0.2}{\sqrt{1 - (0.2)^2}} \frac{2\sqrt{(50)(10)}}{|10 - 50|}\right] \\
&= 0.29, \text{ or } 29\%.
\end{aligned}$$

Simulation

- ADS Simulation

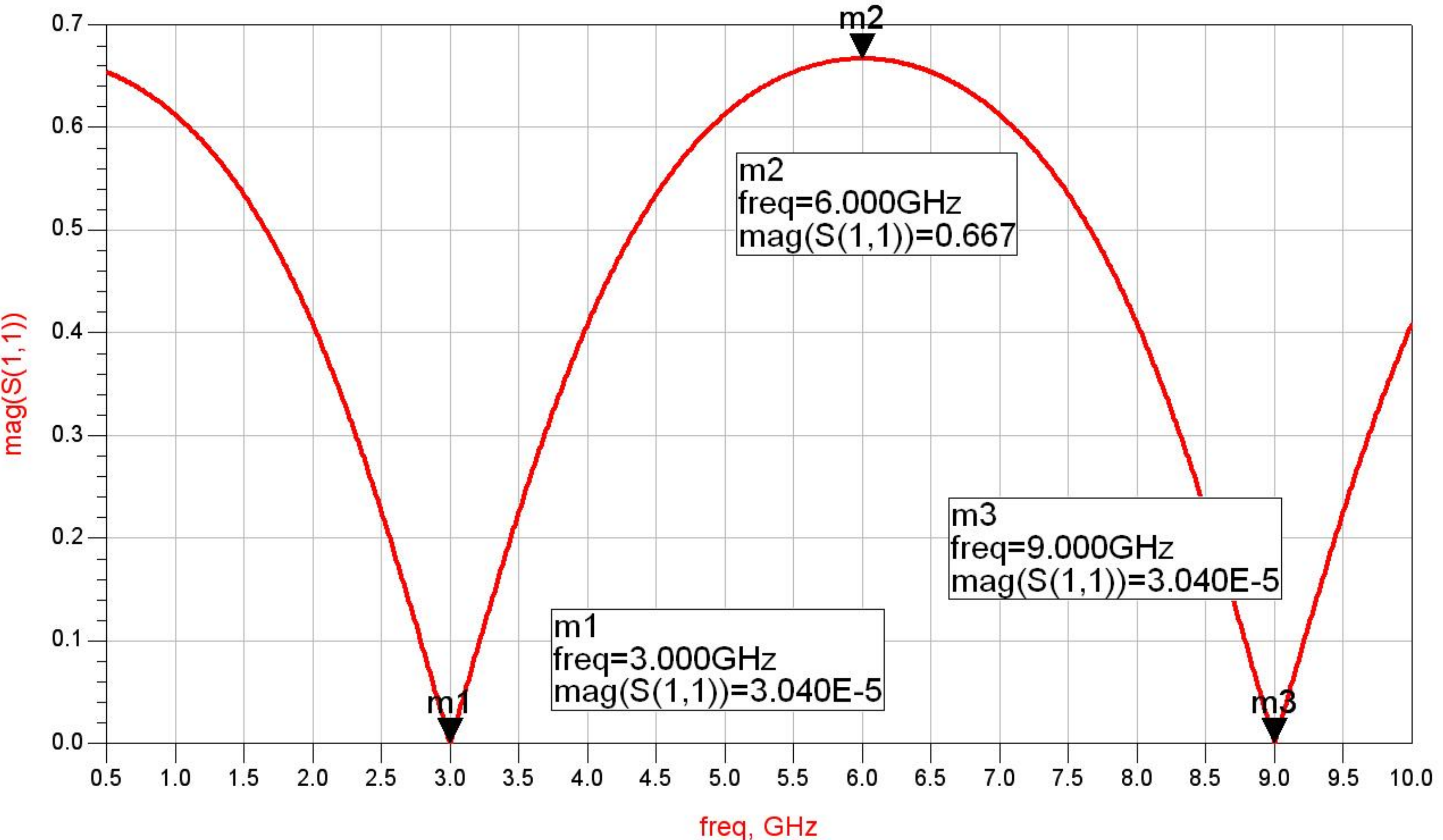


$$\Delta f = 0.88GHz$$

$$\left|\Gamma\left(3GHz\right)\right| = 3\cdot 10^{-5}$$

$$\frac{\Delta f}{f_0} = \frac{0.88}{3} = 0.2933$$

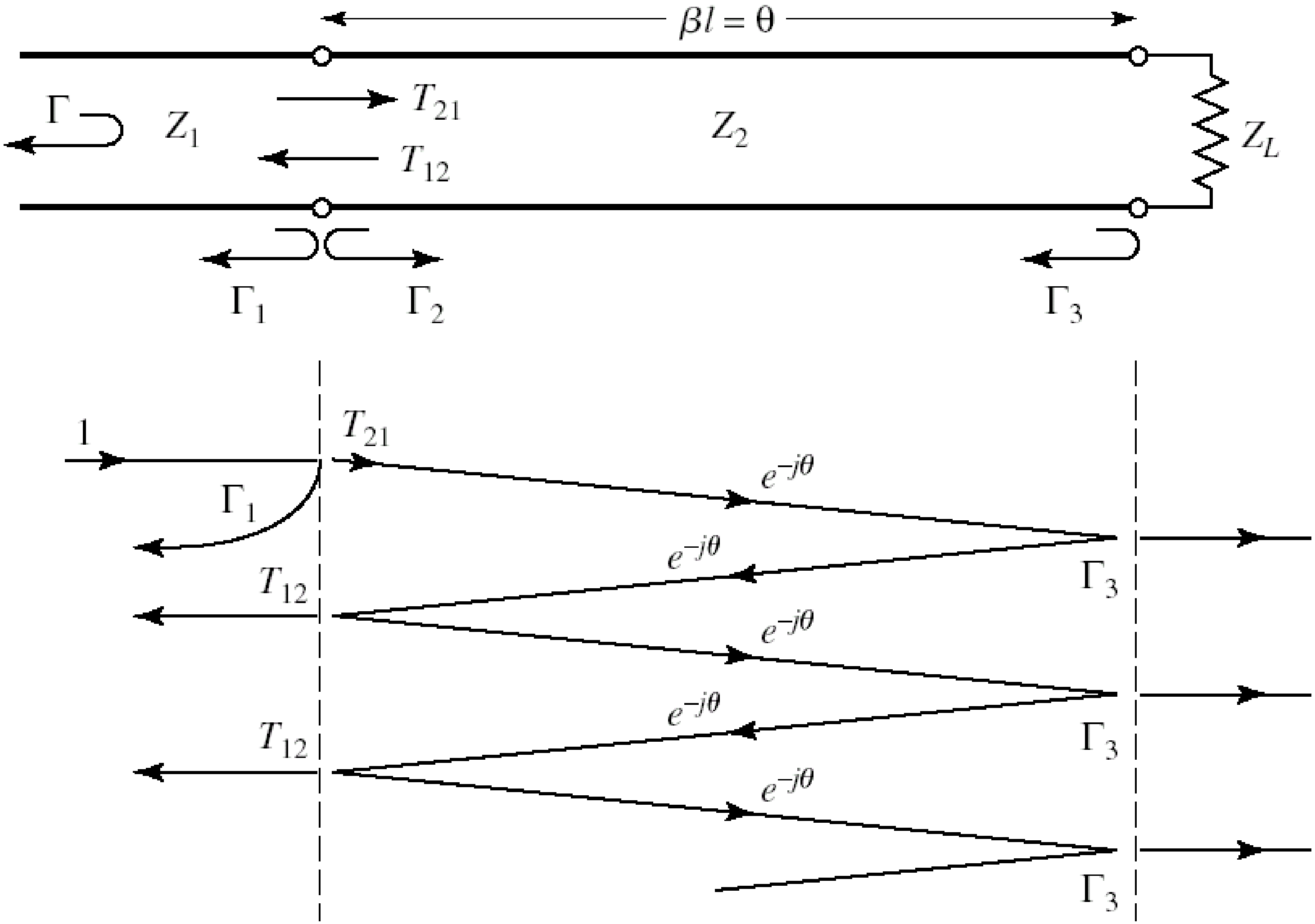
Full bandwidth simulation



Multisection Impedance Transformer

- The quarter-wave transformer can match any real load to any feed line impedance
- If a greater bandwidth for the match is required we must use multiple sections of transmission lines transformers:
  - binomial
  - Chebyshev

The theory of small reflections



The theory of small reflections

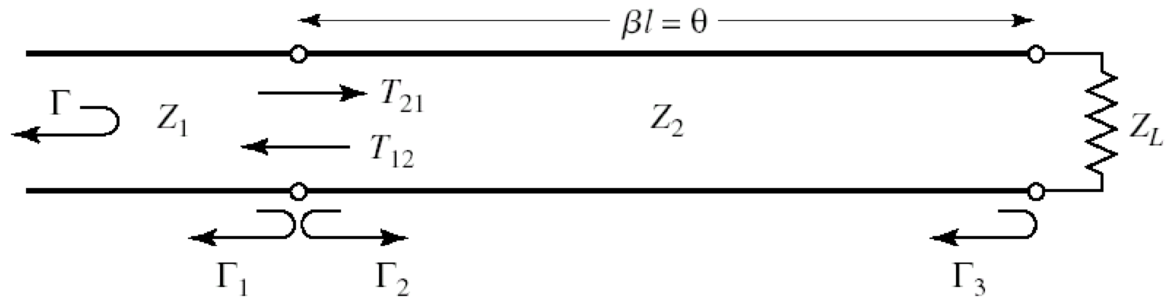
$$\Gamma_1 = \frac{Z_2 - Z_1}{Z_2 + Z_1}$$

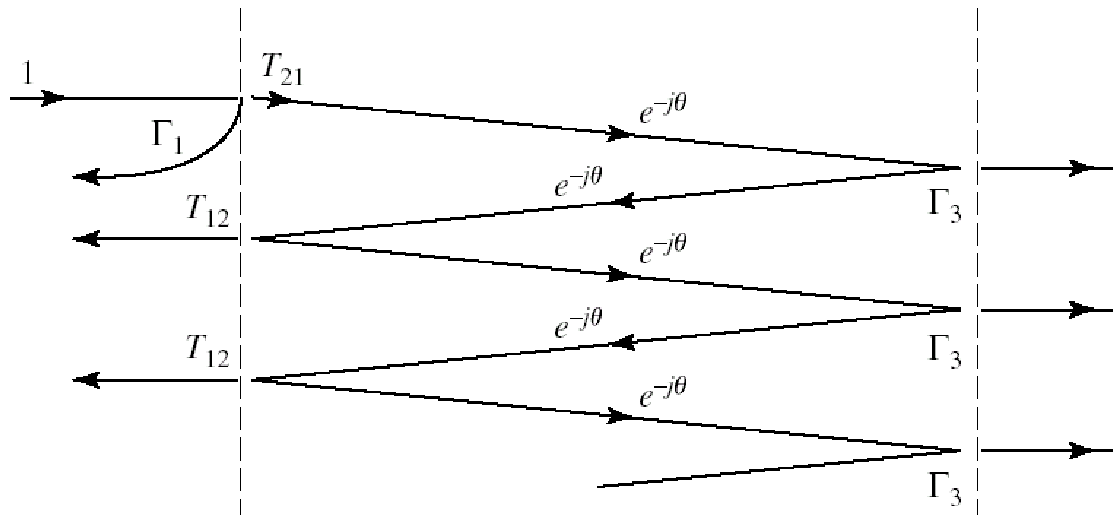
$$\Gamma_2 = -\Gamma_1$$

$$\Gamma_3 = \frac{Z_L - Z_2}{Z_L + Z_2}$$

$$T_{21} = 1 + \Gamma_1 = \frac{2 \cdot Z_2}{Z_1 + Z_2}$$

$$T_{12} = 1 + \Gamma_2 = \frac{2 \cdot Z_1}{Z_1 + Z_2}$$





$$\Gamma = \Gamma_1 + T_{12} \cdot T_{21} \cdot \Gamma_3 \cdot e^{-2j\theta} + T_{12} \cdot T_{21} \cdot \Gamma_3^2 \cdot \Gamma_2 \cdot e^{-4j\theta} + T_{12} \cdot T_{21} \cdot \Gamma_3^3 \cdot \Gamma_2^2 \cdot e^{-6j\theta} + \cdots$$

$$\Gamma = \Gamma_1 + T_{12} \cdot T_{21} \cdot \Gamma_3 \cdot e^{-2j\theta} \sum_{n=0}^{\infty} \Gamma_3^n \cdot \Gamma_2^n \cdot e^{-2jn\theta}$$

The theory of small reflections

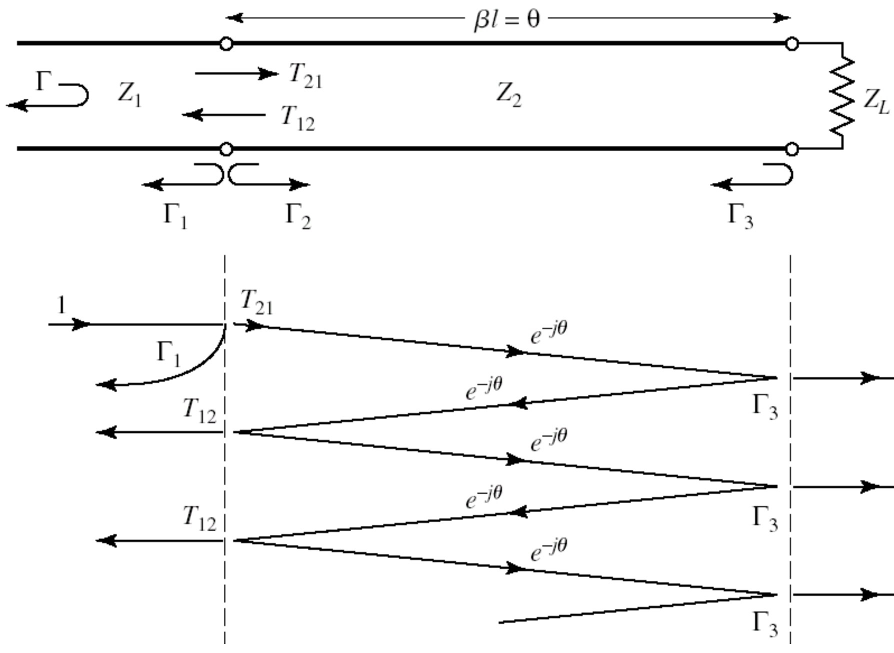
$$\Gamma = \Gamma_1 + T_{12} \cdot T_{21} \cdot \Gamma_3 \cdot e^{-2j\theta} \sum_{n=0}^{\infty} \Gamma_3^n \cdot \Gamma_2^n \cdot e^{-2jn\theta}$$

$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x} \qquad |x| < 1$$

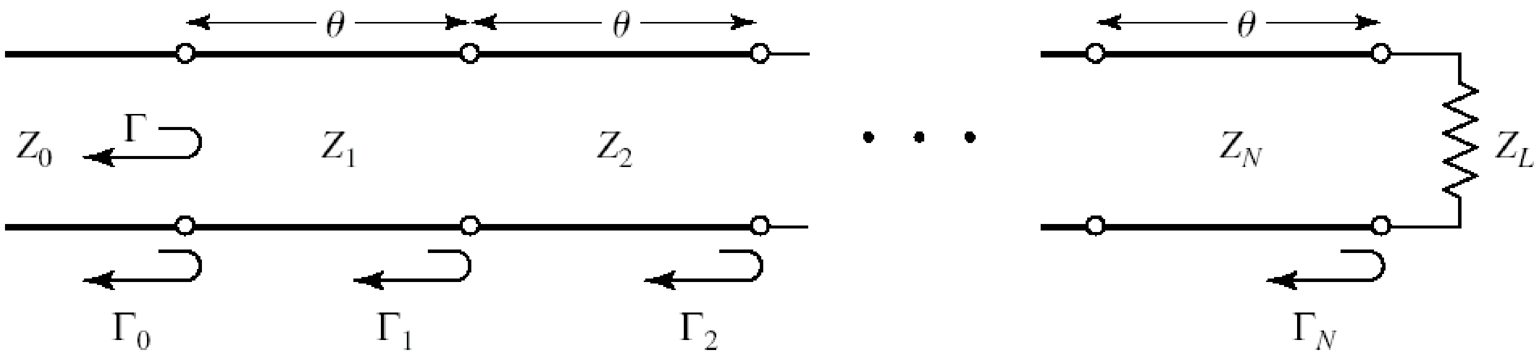
$$\Gamma = \frac{\Gamma_1 + \Gamma_3 \cdot e^{-2j\theta}}{1 + \Gamma_1 \cdot \Gamma_3 \cdot e^{-2j\theta}}$$

- If the discontinuities between the impedances $Z_1 \div Z_2$ and $Z_2 \div Z_L$ are small we can approximate

$$\Gamma \cong \Gamma_1 + \Gamma_3 \cdot e^{-2j\theta}$$



Multisection transformers



$$\Gamma_1 = \frac{Z_1 - Z_0}{Z_1 + Z_0}$$

$$\Gamma_n = \frac{Z_{n+1} - Z_n}{Z_{n+1} + Z_n}$$

$$n = \overline{1, N-1}$$

$$\Gamma_N = \frac{Z_L - Z_N}{Z_L + Z_N}$$

- We also assume that all impedances **increase or decrease monotonically** across the transformer
- This implies that all reflection coefficients will be real and of the same sign
- Previously, 1 section $\Gamma \cong \Gamma_1 + \Gamma_3 \cdot e^{-2j\theta} \Rightarrow$

$$\Gamma(\theta) = \Gamma_0 + \Gamma_1 \cdot e^{-2j\theta} + \Gamma_2 \cdot e^{-4j\theta} + \cdots + \Gamma_N \cdot e^{-2jN\theta}$$

Multisection transformers

- assume that the transformer can be made **symmetrical**

$$\Gamma_0 = \Gamma_N, \Gamma_1 = \Gamma_{N-1}, \Gamma_2 = \Gamma_{N-2} \cdots$$

- Note that this does **not** imply that the impedances are symmetrical

$$\Gamma(\theta) = \Gamma_0 + \Gamma_1 \cdot e^{-2j\theta} + \Gamma_2 \cdot e^{-4j\theta} + \cdots + \Gamma_N \cdot e^{-2jN\theta}$$

$$\Gamma(\theta) = e^{-jN\theta} \cdot \left[\Gamma_0 \cdot \left(e^{jN\theta} + e^{-jN\theta}\right) + \Gamma_1 \cdot \left(e^{j(N-2)\theta} + e^{-j(N-2)\theta}\right) + \Gamma_2 \cdot \left(e^{j(N-4)\theta} + e^{-j(N-4)\theta}\right) + \cdots\right]$$

$$\Gamma(\theta) = 2e^{-jN\theta} \cdot \left[\Gamma_0 \cdot \cos N\theta + \Gamma_1 \cdot \cos(N-2)\theta + \cdots \Gamma_n \cdot \cos(N-2n)\theta + \cdots\right]$$

last item: $\cdots \frac{1}{2} \cdot \Gamma_{N/2} \quad N \text{ even}$ $\qquad \cdots \Gamma_{(N-1)/2} \cdot \cos\theta \quad N \text{ odd}$

Multisection transformers

- Input reflection coefficient

$$\Gamma(\theta) = \Gamma_0 + \Gamma_1 \cdot e^{-2j\theta} + \Gamma_2 \cdot e^{-4j\theta} + \cdots + \Gamma_N \cdot e^{-2jN\theta}$$

$$e^{-2j\theta} \equiv x$$

$$f(x) = a_0 + a_1 \cdot x + a_2 \cdot x^2 + \cdots + a_N \cdot x^N$$

- we can choose the coefficients so we obtain a desired behavior (of the polynomial)

Binomial multisection transformer

- The response is as flat as possible near the design frequency, also known as **maximally flat**
- For N sections the first N-1 derivatives of the $|\Gamma(\theta)|$ functions are annuled

$$f(x) = A \cdot (1+x)^N$$

$$\Gamma(\theta) = A \cdot \left(1 + e^{-2j\theta}\right)^N$$

$$|\Gamma(\theta)| = |A| \cdot \left|e^{-j\theta}\right|^N \cdot \left|e^{j\theta} + e^{-j\theta}\right|^N = 2^N \cdot |A| \cdot |\cos\theta|^N$$

$$\left|\Gamma\left(\frac{\pi}{2}\right)\right| = 0; \quad \frac{d^n}{d\theta^n}|\Gamma(\theta)|\Big|_{\theta=\frac{\pi}{2}} = 0 \qquad n = \overline{1, N-1} \qquad l = \frac{\lambda}{4} \Rightarrow \theta = \beta \cdot l = \frac{\pi}{2}$$

Binomial multisection transformer

- A, θ →0 , 0 length sections, the sections disappear

$$\Gamma(0) = 2^N \cdot A = \frac{Z_L - Z_0}{Z_L + Z_0} \qquad A = 2^{-N} \cdot \frac{Z_L - Z_0}{Z_L + Z_0}$$

- Binomial expansion

$$f(x) = (1+x)^N = C_N^0 + C_N^1 \cdot x + \cdots + C_N^n \cdot x^n + \cdots + C_N^N \cdot x^N$$

$$C_N^n = \frac{N!}{(N-n)!\,n!}$$

- Reflection coefficient:

$$\Gamma(\theta) = A \cdot \left(1 + e^{-2j\theta}\right)^N \qquad \Gamma(\theta) = \Gamma_0 + \Gamma_1 \cdot e^{-2j\theta} + \Gamma_2 \cdot e^{-4j\theta} + \cdots + \Gamma_N \cdot e^{-2jN\theta}$$

$$\Gamma_n = A \cdot C_N^n$$

Binomial multisection transformer

- Manual design procedure

$$A = 2^{-N} \cdot \frac{Z_L - Z_0}{Z_L + Z_0} \qquad \Gamma_n = A \cdot C_N^n$$

$$\Gamma_n = \frac{Z_{n+1} - Z_n}{Z_{n+1} + Z_n} \cong \frac{1}{2} \ln \frac{Z_{n+1}}{Z_n} \qquad \ln x \cong 2 \cdot \frac{x-1}{x+1} \quad x \cong 1$$

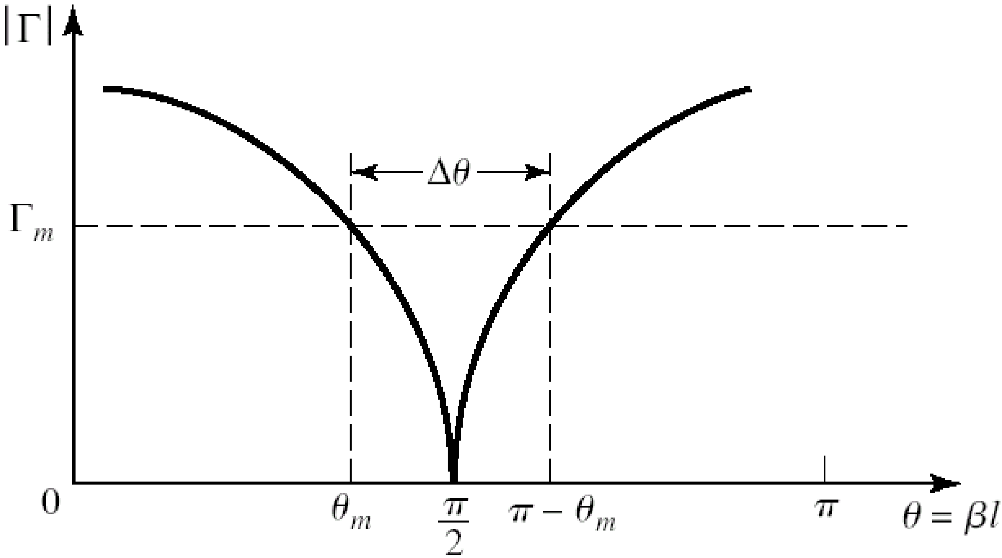
$$\ln \frac{Z_{n+1}}{Z_n} \cong 2 \cdot \Gamma_n = 2 \cdot A \cdot C_N^n = 2 \cdot 2^{-N} \cdot \frac{Z_L - Z_0}{Z_L + Z_0} \cong 2^{-N} \cdot C_N^n \cdot \ln \frac{Z_L}{Z_0}$$

$$\ln Z_{n+1} \cong \ln Z_n + 2^{-N} \cdot C_N^n \cdot \ln \frac{Z_L}{Z_0}$$

Binomial multisection transformer

- Bandwidth, Γ_m maximum acceptable value

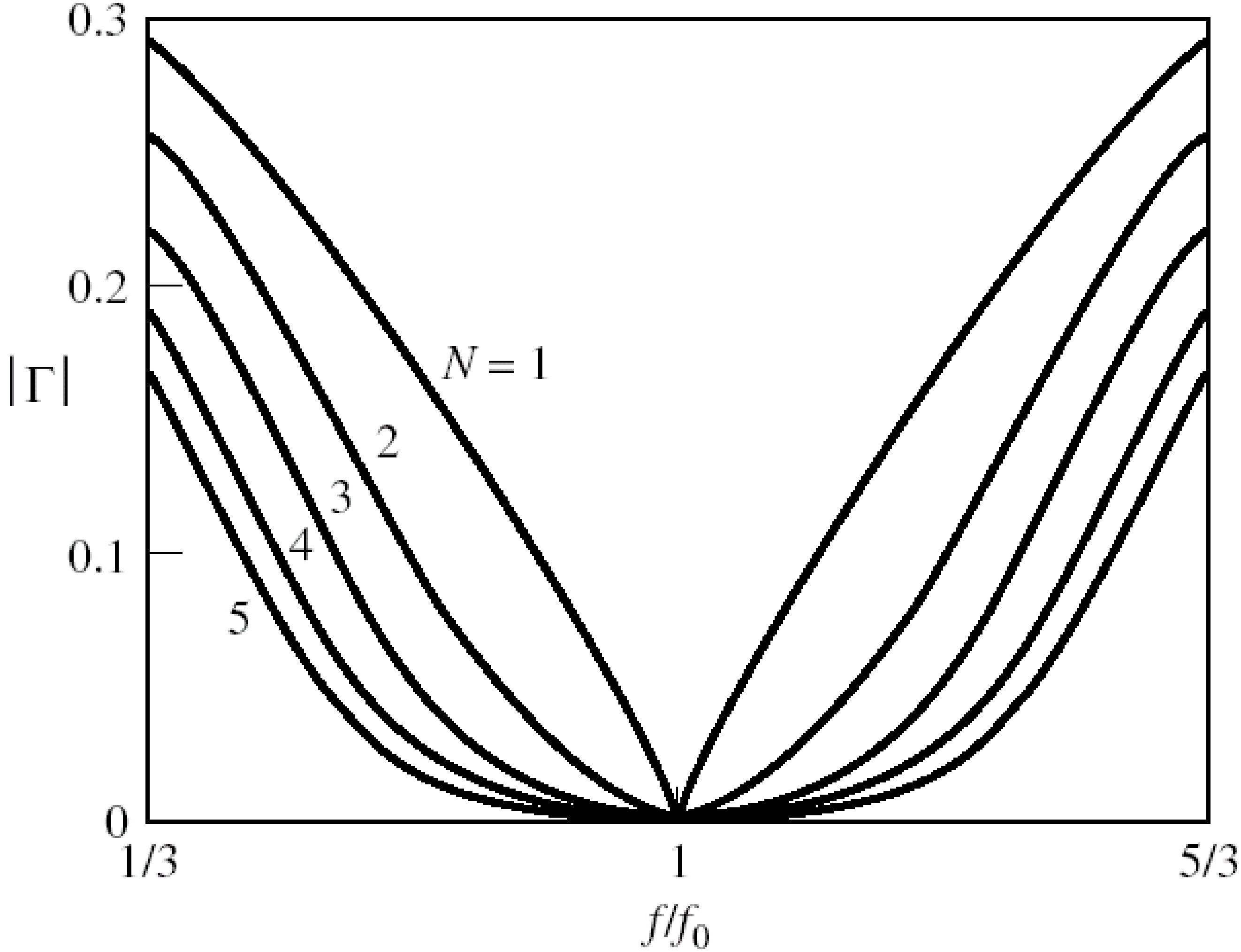
$$\Gamma_m = |\Gamma(\theta_m)| = 2^N \cdot |A| \cdot |\cos\theta_m|^N$$



$$\theta_m = \cos^{-1}\left[\frac{1}{2}\left(\frac{\Gamma_m}{|A|}\right)^{\frac{1}{N}}\right]$$

$$\frac{\Delta f}{f_0} = \frac{2(f_0 - f_m)}{f_0} = 2 - \frac{4\theta_m}{\pi} = 2 - \frac{4}{\pi} \cdot \cos^{-1}\left[\frac{1}{2}\left(\frac{\Gamma_m}{|A|}\right)^{\frac{1}{N}}\right]$$

Bandwidth



Binomial multisection transformer
Exact results

Z_L/Z_0	N = 2 Z_1/Z_0	N = 2 Z_2/Z_0	N = 3 Z_1/Z_0	N = 3 Z_2/Z_0	N = 3 Z_3/Z_0	N = 4 Z_1/Z_0	N = 4 Z_2/Z_0	N = 4 Z_3/Z_0	N = 4 Z_4/Z_0
1.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1.5	1.1067	1.3554	1.0520	1.2247	1.4259	1.0257	1.1351	1.3215	1.4624
2.0	1.1892	1.6818	1.0907	1.4142	1.8337	1.0444	1.2421	1.6102	1.9150
3.0	1.3161	2.2795	1.1479	1.7321	2.6135	1.0718	1.4105	2.1269	2.7990
4.0	1.4142	2.8285	1.1907	2.0000	3.3594	1.0919	1.5442	2.5903	3.6633
6.0	1.5651	3.8336	1.2544	2.4495	4.7832	1.1215	1.7553	3.4182	5.3500
8.0	1.6818	4.7568	1.3022	2.8284	6.1434	1.1436	1.9232	4.1597	6.9955
10.0	1.7783	5.6233	1.3409	3.1623	7.4577	1.1613	2.0651	4.8424	8.6110

Z_L/Z_0	N = 5 Z_1/Z_0	N = 5 Z_2/Z_0	N = 5 Z_3/Z_0	N = 5 Z_4/Z_0	N = 5 Z_5/Z_0	N = 6 Z_1/Z_0	N = 6 Z_2/Z_0	N = 6 Z_3/Z_0	N = 6 Z_4/Z_0	N = 6 Z_5/Z_0	N = 6 Z_6/Z_0
1.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1.5	1.0128	1.0790	1.2247	1.3902	1.4810	1.0064	1.0454	1.1496	1.3048	1.4349	1.4905
2.0	1.0220	1.1391	1.4142	1.7558	1.9569	1.0110	1.0790	1.2693	1.5757	1.8536	1.9782
3.0	1.0354	1.2300	1.7321	2.4390	2.8974	1.0176	1.1288	1.4599	2.0549	2.6577	2.9481
4.0	1.0452	1.2995	2.0000	3.0781	3.8270	1.0225	1.1661	1.6129	2.4800	3.4302	3.9120
6.0	1.0596	1.4055	2.4495	4.2689	5.6625	1.0296	1.2219	1.8573	3.2305	4.9104	5.8275
8.0	1.0703	1.4870	2.8284	5.3800	7.4745	1.0349	1.2640	2.0539	3.8950	6.3291	7.7302
10.0	1.0789	1.5541	3.1623	6.4346	9.2687	1.0392	1.2982	2.2215	4.5015	7.7030	9.6228

Exemple

- Design a three-section binomial transformer to match a 30Ω load to a 100 Ω line at f_o=3GHz, Γ_m=0.1
  - N = 3

$$Z_L = 30\Omega \qquad Z_0 = 100\Omega$$

$$A = 2^{-N}\frac{Z_L - Z_0}{Z_L + Z_0} \approx \frac{1}{2^{N+1}}\ln\frac{Z_L}{Z_0} = -0.07525$$

$$C_3^0 = \frac{3!}{3!\cdot 0!} = 1 \qquad C_3^1 = \frac{3!}{2!\cdot 1!} = 3 \qquad C_3^2 = \frac{3!}{1!\cdot 2!} = 3$$

Exemple

$$n = 0$$

$$\ln Z_1 = \ln Z_0 + 2^{-N} C_3^0 \ln \frac{Z_L}{Z_0} = \ln 100 + 2^{-3} \cdot 1 \cdot \ln \frac{30}{100} = 4.455$$

$$Z_1 = 86.03\Omega$$

$$n = 1$$

$$\ln Z_2 = \ln Z_1 + 2^{-N} C_3^1 \ln \frac{Z_L}{Z_0} = \ln 86.03 + 2^{-3} \cdot 3 \cdot \ln \frac{30}{100} = 4.003$$

$$Z_2 = 54.77\Omega$$

$$n = 2$$

$$\ln Z_3 = \ln Z_2 + 2^{-N} C_3^2 \ln \frac{Z_L}{Z_0} = \ln 54.77 + 2^{-3} \cdot 3 \cdot \ln \frac{30}{100} = 3.552$$

$$Z_3 = 34.87\Omega$$

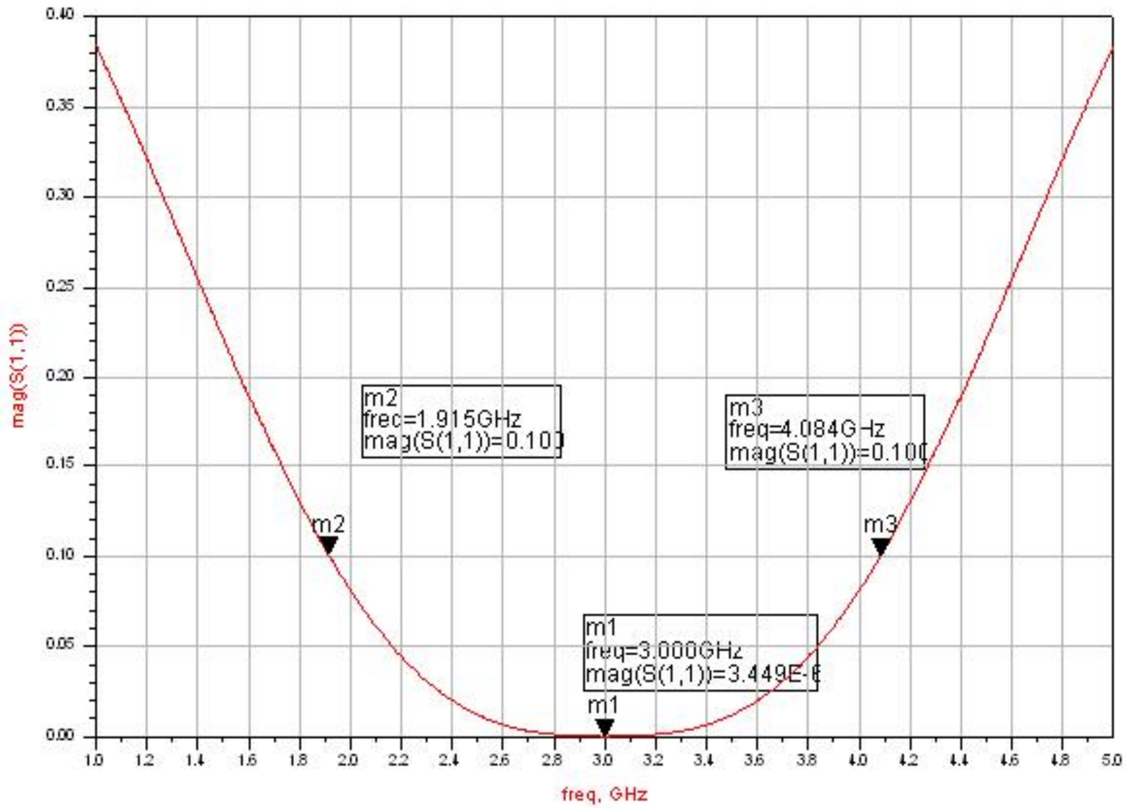
Exemple

$$\frac{\Delta f}{f_0} = 2 - \frac{4}{\pi}\arccos\left[\frac{1}{2}\left(\frac{\Gamma_m}{|A|}\right)^{1/N}\right] = 2 - \frac{4}{\pi}\arccos\left[\frac{1}{2}\left(\frac{0.1}{0.07525}\right)^{1/3}\right] = 0.74$$

$$\Delta f = 2.22GHz$$

Simulation

- Similarly Lab. 1



$$\Delta f = 2.169 GHz$$

$$|\Gamma(3GHz)| = 3.5 \cdot 10^{-6}$$

Chebyshev multisection transformer

- The response of this multisection impedance transformer is equal-ripple in passband
- optimizes (increases) bandwidth at the expense of passband ripple
- We match the $\Gamma(\theta)$ function with an desired Chebyshev polynomial

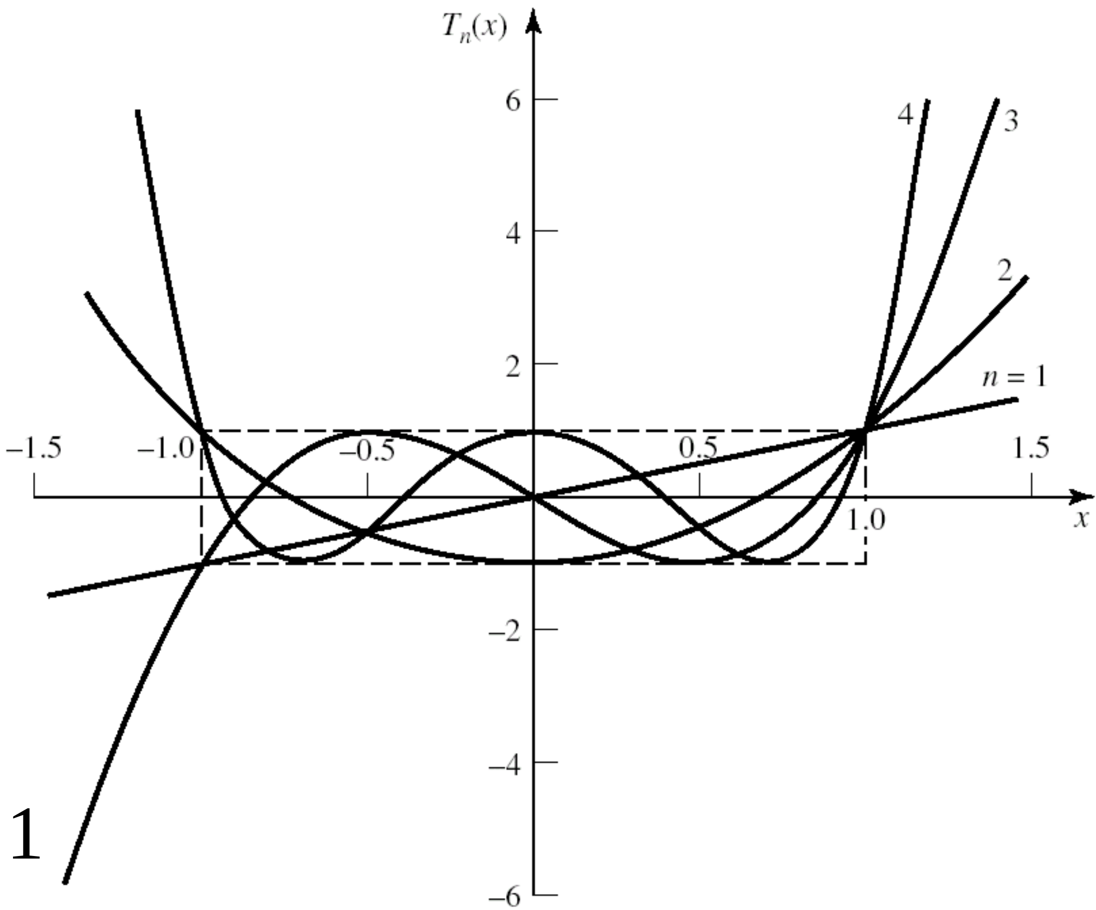
Chebyshev polynomials

$$T_1(x) = x$$

$$T_2(x) = 2x^2 - 1$$

$$T_3(x) = 4x^3 - 3x$$

$$T_4(x) = 8x^4 - 8x^2 + 1$$



- equal-ripple

$$-1 \le x \le 1 \quad \Rightarrow \quad |T_n(x)| \le 1$$

$$T_n(x) = 2xT_{n-1}(x) - T_{n-2}(x)$$

Chebyshev polynomials

$$\Gamma(\theta) = \Gamma_0 + \Gamma_1 \cdot e^{-2j\theta} + \Gamma_2 \cdot e^{-4j\theta} + \cdots + \Gamma_N \cdot e^{-2jN\theta}$$

$$e^{-2j\theta} \equiv x$$

$$f(x) = a_0 + a_1 \cdot x + a_2 \cdot x^2 + \cdots + a_N \cdot x^N$$

$$\Gamma(\theta) = 2e^{-jN\theta} \cdot \left[\Gamma_0 \cdot \cos N\theta + \Gamma_1 \cdot \cos(N-2)\theta + \cdots \Gamma_n \cdot \cos(N-2n)\theta + \cdots\right]$$

last item:

$$\cdots \frac{1}{2} \cdot \Gamma_{N/2} \quad N \text{ even}$$

$$\cdots \Gamma_{(N-1)/2} \cdot \cos\theta \quad N \text{ odd}$$

$$x = \cos\theta \qquad |x| < 1$$

- We can show that: $T_n(\cos\theta) = \cos(n\theta)$

$$T_n(x) = \cos(n \arccos(x)) \quad |x| < 1 \qquad T_n(x) = \cosh\left(n \cosh^{-1}(x)\right) \quad |x| > 1$$

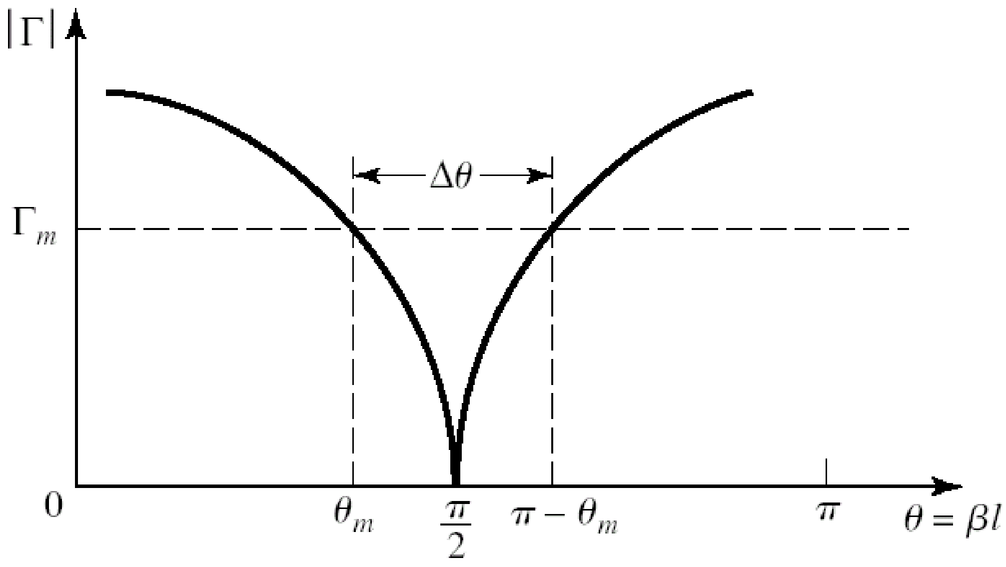
Chebyshev multisection transformer

- variable change so we map:

$$\theta = \theta_m \rightarrow x = 1$$

$$\theta = \pi - \theta_m \rightarrow x = -1$$

$$x \equiv \frac{\cos\theta}{\cos\theta_m}$$



$$\sec\theta = \frac{1}{\cos\theta}$$

$$x = \sec\theta_m \cos\theta$$

Chebyshev multisection transformer

$$T_1(\sec\theta_m \cos\theta) = \sec\theta_m \cos\theta$$

$$T_2(\sec\theta_m \cos\theta) = \sec^2\theta_m (1+\cos 2\theta) - 1$$

$$T_3(\sec\theta_m \cos\theta) = \sec^3\theta_m (\cos 3\theta + 3\cos\theta) - 3\sec\theta_m \cos\theta$$

$$T_4(\sec\theta_m \cos\theta) = \sec^4\theta_m (\cos 4\theta + 4\cos 2\theta + 3) - 4\sec^2\theta_m (\cos 2\theta + 1) + 1$$

- We search coefficients of Γ(θ) function to obtain a Chebyshev polynomial

$$\Gamma(\theta) = 2e^{-jN\theta} \cdot [\Gamma_0 \cdot \cos N\theta + \Gamma_1 \cdot \cos(N-2)\theta + \cdots \Gamma_n \cdot \cos(N-2n)\theta + \cdots]$$

$$\Gamma(\theta) = A \cdot e^{-jN\theta} \cdot T_N(\sec\theta_m \cos\theta)$$

last item:

$$\cdots \frac{1}{2} \cdot \Gamma_{N/2} \quad N \text{ even}$$

$$\cdots \Gamma_{(N-1)/2} \cdot \cos\theta \quad N \text{ odd}$$

Chebyshev multisection transformer

- A, θ →0 , 0 length sections, the sections disappear

$$\Gamma(0)=\frac{Z_L-Z_0}{Z_L+Z_0}=A\cdot T_N(\sec\theta_m) \qquad A=\frac{Z_L-Z_0}{Z_L+Z_0}\cdot\frac{1}{T_N(\sec\theta_m)} \qquad \Gamma_m=|A|$$

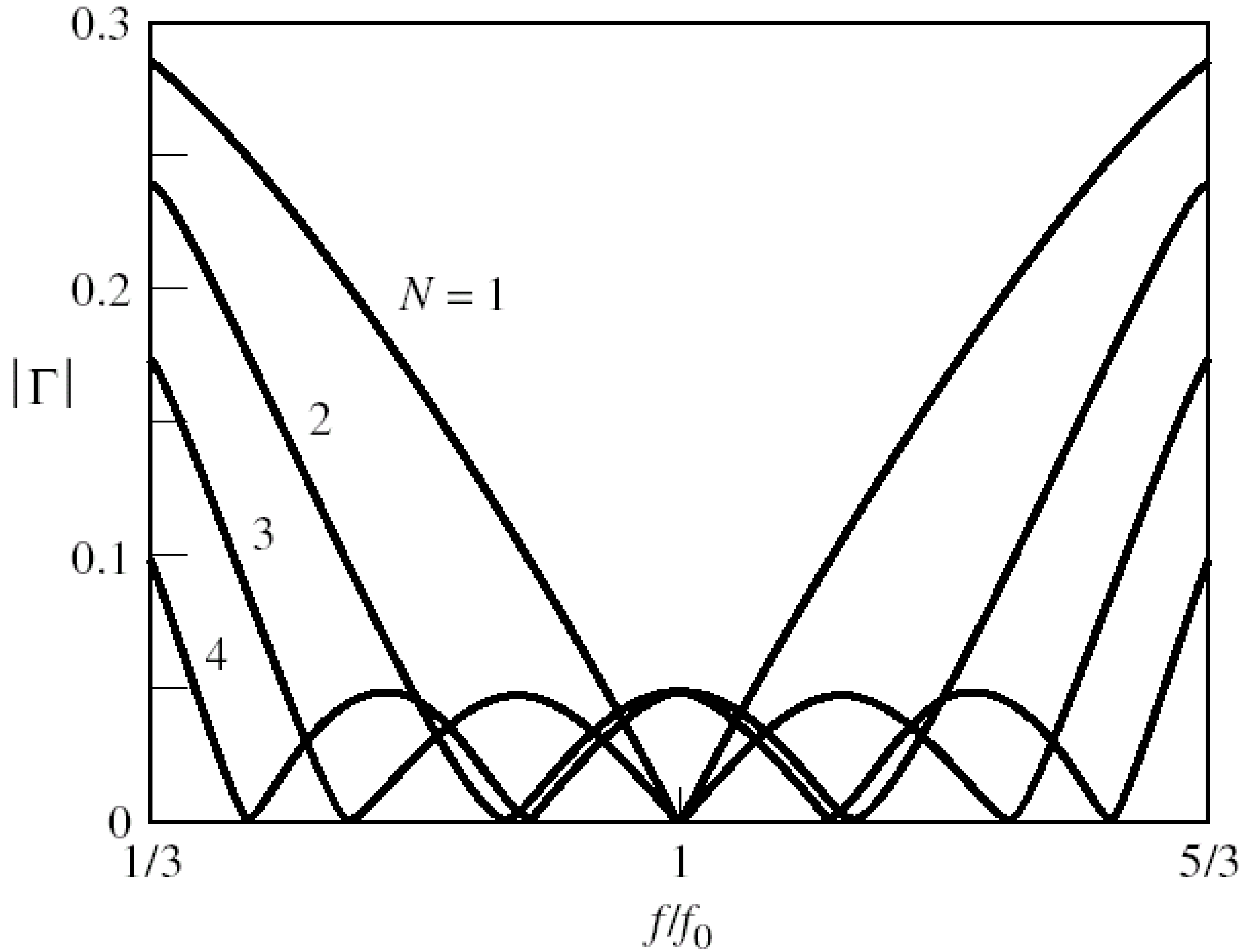
$$T_N(\sec\theta_m)=\frac{1}{\Gamma_m}\left|\frac{Z_L-Z_0}{Z_L+Z_0}\right|\cong\frac{1}{2\Gamma_m}\left|\ln\frac{Z_L}{Z_0}\right|$$

$$T_n(x)=\cosh\left(n\cosh^{-1}(x)\right)$$

$$\sec\theta_m=\cosh\left[\frac{1}{N}\cosh^{-1}\left(\frac{1}{\Gamma_m}\left|\frac{Z_L-Z_0}{Z_L+Z_0}\right|\right)\right]\cong\cosh\left[\frac{1}{N}\cosh^{-1}\left(\left|\frac{\ln(Z_L/Z_0)}{2\Gamma_m}\right|\right)\right]$$

$$\frac{\Delta f}{f_0}=\frac{2(f_0-f_m)}{f_0}=2-\frac{4\theta_m}{\pi}$$

Bandwidth



Chebyshev multisection transformer
Exact results

	$N = 2$				$N = 3$					
	$\Gamma_m = 0.05$		$\Gamma_m = 0.20$		$\Gamma_m = 0.05$			$\Gamma_m = 0.20$		
Z_L/Z_0	Z_1/Z_0	Z_2/Z_0	Z_1/Z_0	Z_2/Z_0	Z_1/Z_0	Z_2/Z_0	Z_3/Z_0	Z_1/Z_0	Z_2/Z_0	Z_3/Z_0
1.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1.5	1.1347	1.3219	1.2247	1.2247	1.1029	1.2247	1.3601	1.2247	1.2247	1.2247
2.0	1.2193	1.6402	1.3161	1.5197	1.1475	1.4142	1.7429	1.2855	1.4142	1.5558
3.0	1.3494	2.2232	1.4565	2.0598	1.2171	1.7321	2.4649	1.3743	1.7321	2.1829
4.0	1.4500	2.7585	1.5651	2.5558	1.2662	2.0000	3.1591	1.4333	2.0000	2.7908
6.0	1.6047	3.7389	1.7321	3.4641	1.3383	2.4495	4.4833	1.5193	2.4495	3.9492
8.0	1.7244	4.6393	1.8612	4.2983	1.3944	2.8284	5.7372	1.5766	2.8284	5.0742
10.0	1.8233	5.4845	1.9680	5.0813	1.4385	3.1623	6.9517	1.6415	3.1623	6.0920

$N = 4$

	$\Gamma_m = 0.05$				$\Gamma_m = 0.20$			
Z_L/Z_0	Z_1/Z_0	Z_2/Z_0	Z_3/Z_0	Z_4/Z_0	Z_1/Z_0	Z_2/Z_0	Z_3/Z_0	Z_4/Z_0
1.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1.5	1.0892	1.1742	1.2775	1.3772	1.2247	1.2247	1.2247	1.2247
2.0	1.1201	1.2979	1.5409	1.7855	1.2727	1.3634	1.4669	1.5715
3.0	1.1586	1.4876	2.0167	2.5893	1.4879	1.5819	1.8965	2.0163
4.0	1.1906	1.6414	2.4369	3.3597	1.3692	1.7490	2.2870	2.9214
6.0	1.2290	1.8773	3.1961	4.8820	1.4415	2.0231	2.9657	4.1623
8.0	1.2583	2.0657	3.8728	6.3578	1.4914	2.2428	3.5670	5.3641
10.0	1.2832	2.2268	4.4907	7.7930	1.5163	2.4210	4.1305	6.5950

Exemple

- Design a three-section Chebyshev transformer to match a 30Ω load to a 100 Ω line at f_o=3GHz, Γ_m=0.1
  - N = 3 $\quad Z_L = 30\Omega \quad Z_0 = 100\Omega$

$$\Gamma(\theta) = 2e^{-j3\theta}\left[\Gamma_0 \cos 3\theta + \Gamma_1 \cos\theta\right] = Ae^{-j3\theta}T_3(\sec\theta_m \cos\theta)$$

$$|A| = \Gamma_m = 0.1 \qquad A = \frac{Z_L - Z_0}{Z_L + Z_0} \cdot \frac{1}{T_N(\sec\theta_m)} \qquad Z_L < Z_0 \rightarrow A < 0 \qquad A = -0.1$$

$$\sec\theta_m = \cosh\left[\frac{1}{N} \cdot \cosh^{-1}\left(\left|\frac{\ln Z_L / Z_0}{2\Gamma_m}\right|\right)\right] = \cosh\left[\frac{1}{3} \cdot \cosh^{-1}\left(\left|\frac{\ln(30/100)}{2 \cdot 0.1}\right|\right)\right] = 1.362$$

$$\theta_m = \arccos\left(\frac{1}{\sec\theta_m}\right) = 0.746 rad = 42.76°$$

Exemple

$$2\left[\Gamma_0 \cos 3\theta + \Gamma_1 \cos\theta\right] = A\sec^3\theta_m\left(\cos 3\theta + 3\cos\theta\right) - 3A\sec\theta_m\cos\theta$$

$\cos 3\theta$ $\quad 2\Gamma_0 = A\sec^3\theta_m$ $\quad \Gamma_0 = -0.1263$

$\cos\theta$ $\quad 2\Gamma_1 = 3A\left(\sec^3\theta_m - \sec\theta_m\right)$ $\quad \Gamma_1 = -0.1747$

simetrie: $\Gamma_3 = \Gamma_0; \quad \Gamma_2 = \Gamma_1$

Exemple

$$n = 0$$

$$\ln Z_1 = \ln Z_0 + 2 \cdot \Gamma_0 = \ln 100 - 2 \cdot 0.1263 = 4.353$$

$$\Gamma_0 = -0.1263$$

$$Z_1 = 77.68\Omega$$

$$\Gamma_1 = -0.1747$$

$$n = 1$$

$$\ln Z_2 = \ln Z_1 + 2 \cdot \Gamma_1 = \ln 77.68 - 2 \cdot 0.1747 = 4.003$$

$$Z_2 = 54.77\Omega$$

$$n = 2$$

$$\ln Z_3 = \ln Z_2 + 2 \cdot \Gamma_2 = \ln 54.77 - 2 \cdot 0.1747 = 3.654$$

$$Z_3 = 38.62\Omega$$

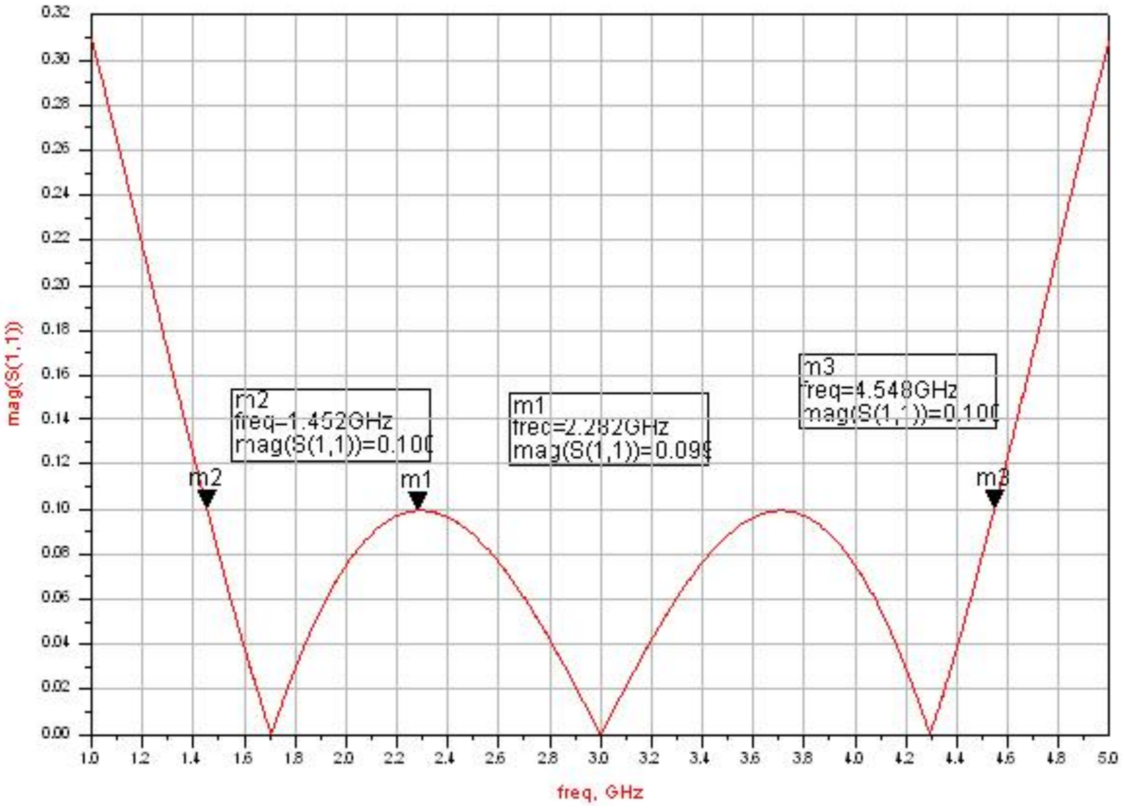
Exemple

$$\frac{\Delta f}{f_0} = \frac{2\left(f_0 - f_m\right)}{f_0} = 2 - \frac{4\theta_m}{\pi} = 2 - \frac{4 \cdot 42.76^\circ}{180^\circ} = 1.045$$

$$\Delta f = 3.15 GHz$$

Simultion

- Similarly Lab. 1



$$\Delta f = 3.096 GHz$$

$$\left|\Gamma\left(3GHz\right)\right| = 4.17 \cdot 10^{-5}$$

$$\left|\Gamma\left(2.282GHz\right)\right| = 0.09925$$

Exact solutions

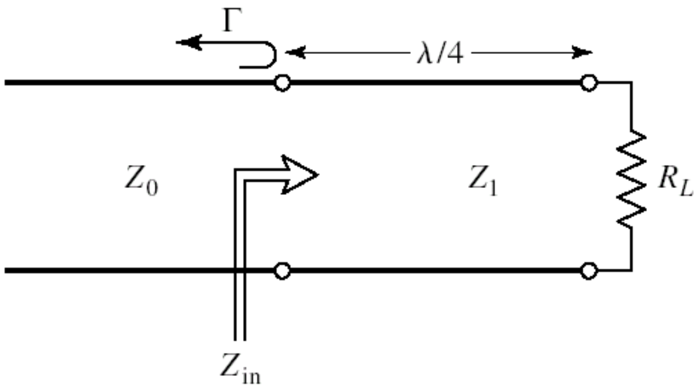
- G. L. Matthaei, L. Young, and E. M. T. Jones, *Microwave Filters, Impedance-Matching Networks,and Coupling Structures, Artech House Books, Dedham, Mass. 1980*

Impedance Matching

Laboratory no. 1

The quarter-wave transformer

- Feed line – input line with characteristic impedance Z_o
- Real load impedance R_L
- We desire matching the load to the fider with a second line with the length λ/4 and characteristic impedance Z_1

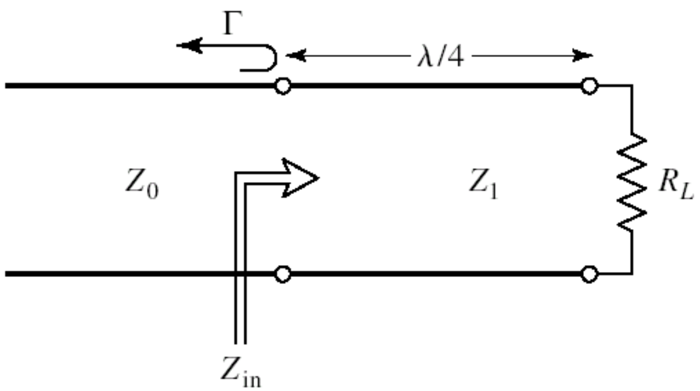


$$Z_{in} = Z_1 \frac{1+\Gamma e^{-2j\beta l}}{1-\Gamma e^{-2j\beta l}}$$

$$\Gamma_O = \frac{V_0^-}{V_0^+} = \frac{R_L - Z_1}{R_L + Z_1}$$

$$Z_{in} = Z_1 \frac{R_L + jZ_1 \tan(\beta l)}{Z_1 + jR_L \tan(\beta l)}$$

The quarter-wave transformer



$$\Gamma_{in} = \frac{Z_{in} - Z_0}{Z_{in} + Z_0}$$

$$\beta \cdot l = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{4} = \frac{\pi}{2} \qquad Z_{in} = \frac{Z_1^2}{R_L}$$

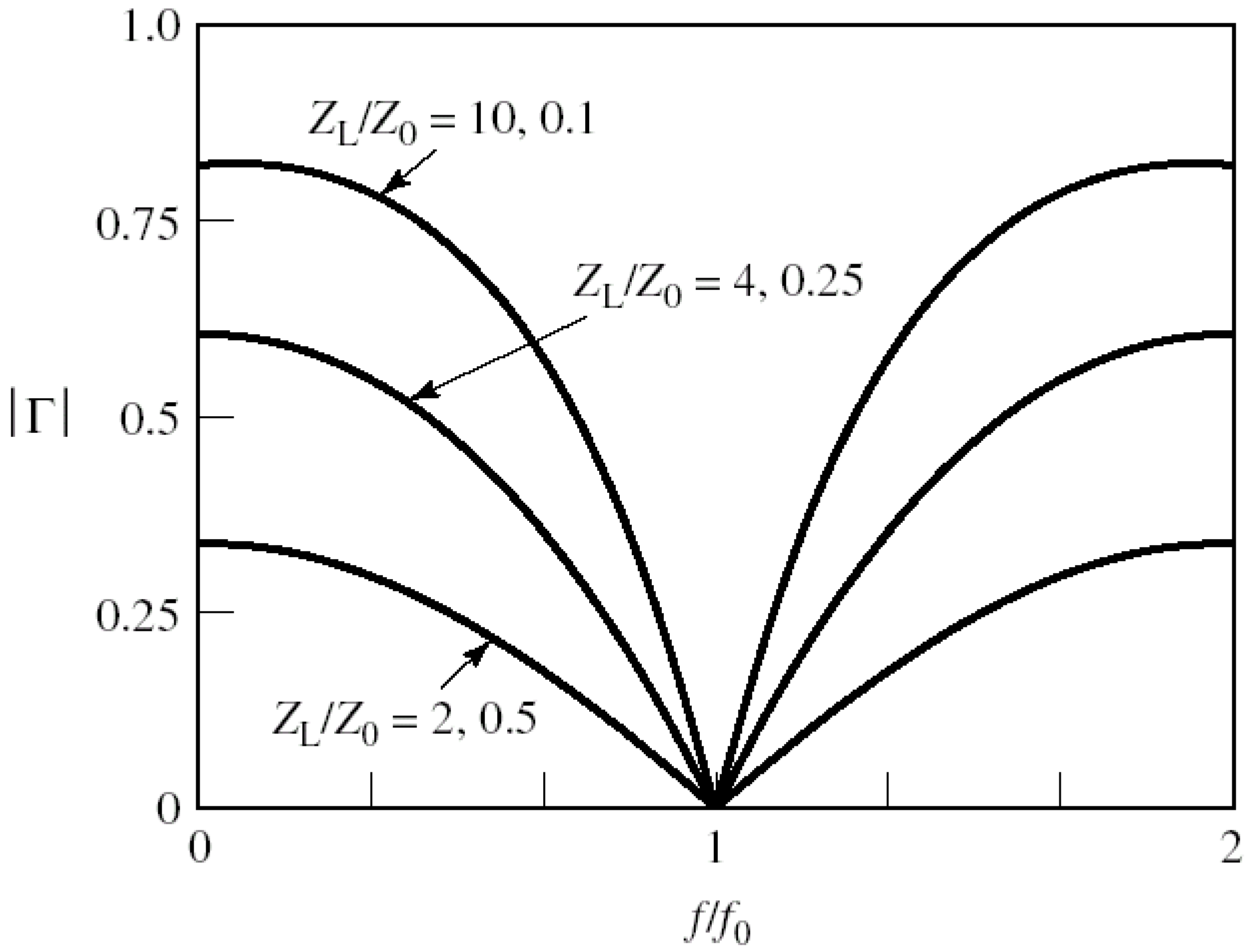
$$\Gamma_{in} = \frac{Z_1^2 - Z_0 \cdot R_L}{Z_1^2 + Z_0 \cdot R_L} \qquad \Gamma_{in} = 0 \quad Z_1 = \sqrt{Z_0 R_L}$$

- In the feed line (Z_o) we have only progressive wave
- In the quarter-wave line (Z_1) we have standing waves

Frequency response

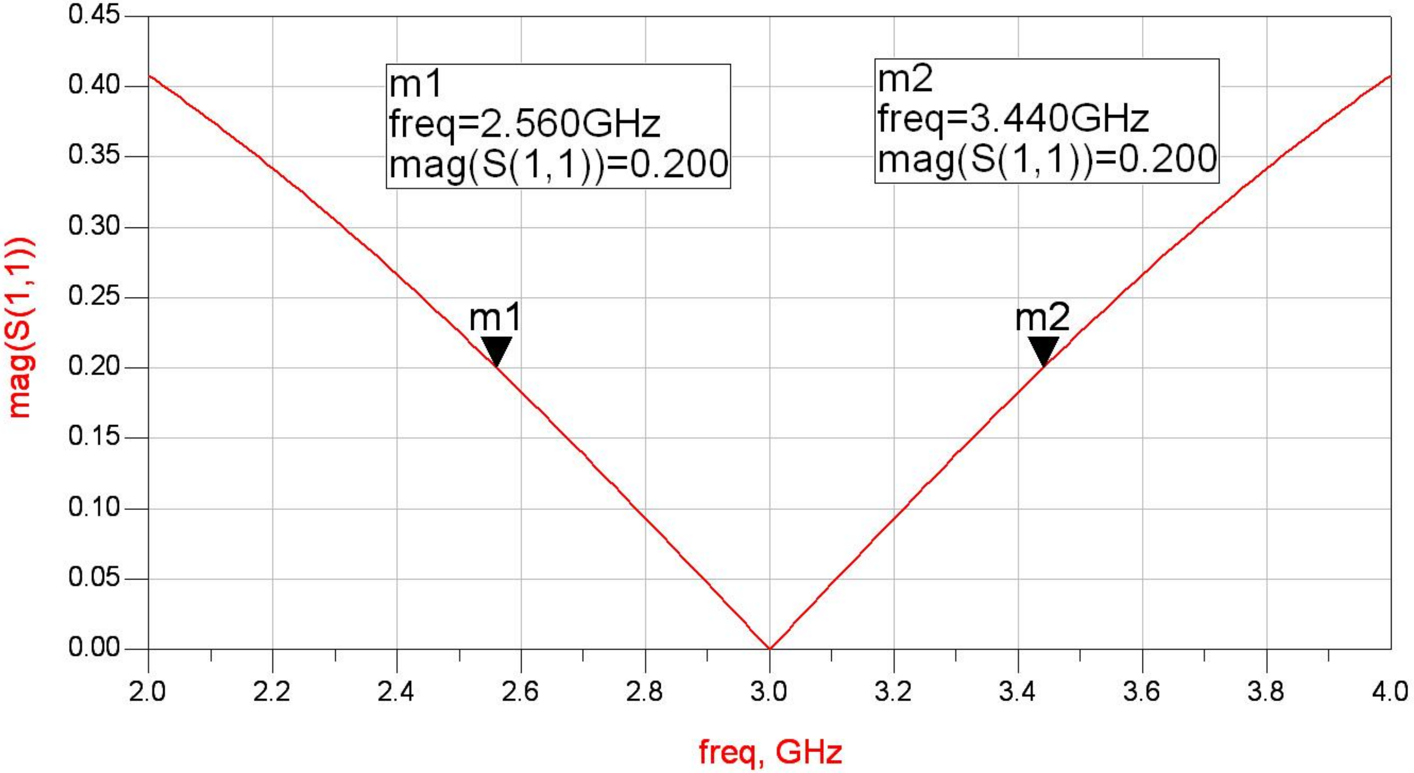
- Bandwidth depends on the initial mismatch

increased bandwidth for smaller load mismatches



Simulation

- ADS Simulation

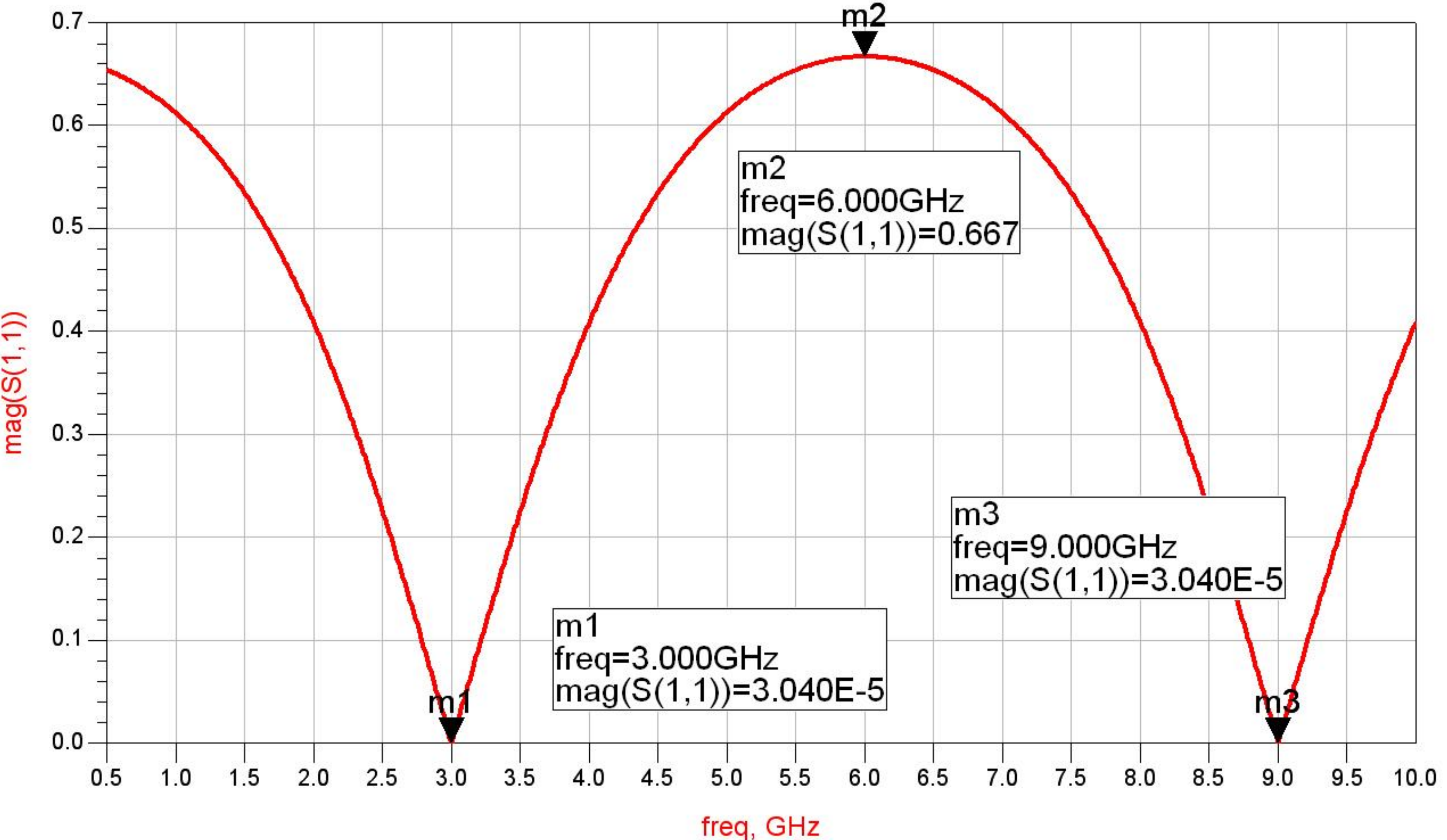


$$\Delta f = 0.88GHz$$

$$|\Gamma(3GHz)| = 3 \cdot 10^{-5}$$

$$\frac{\Delta f}{f_0} = \frac{0.88}{3} = 0.2933$$

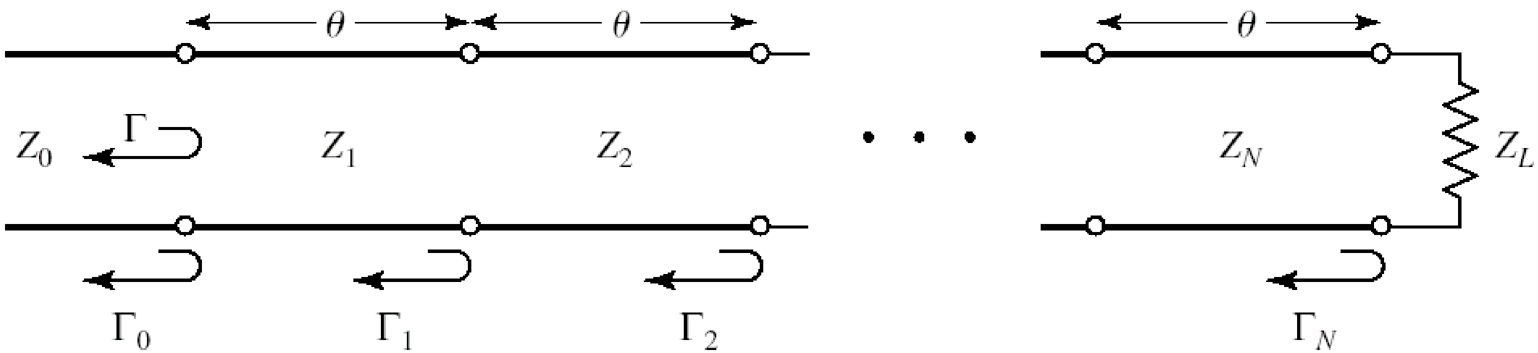
Full bandwidth simulation



Multisection Impedance Transformer

- The quarter-wave transformer can match any real load to any feed line impedance
- If a greater bandwidth for the match is required we must use multiple sections of transmission lines transformers:
  - binomial
  - Chebyshev

Multisection transformers



- We also assume that all impedances **increase or decrease monotonically** across the transformer
- This implies that all reflection coefficients will be real and of the same sign
- Previously, 1 section $\Gamma \cong \Gamma_1 + \Gamma_3 \cdot e^{-2j\theta} \Rightarrow$

$$\Gamma(\theta) = \Gamma_0 + \Gamma_1 \cdot e^{-2j\theta} + \Gamma_2 \cdot e^{-4j\theta} + \cdots + \Gamma_N \cdot e^{-2jN\theta}$$

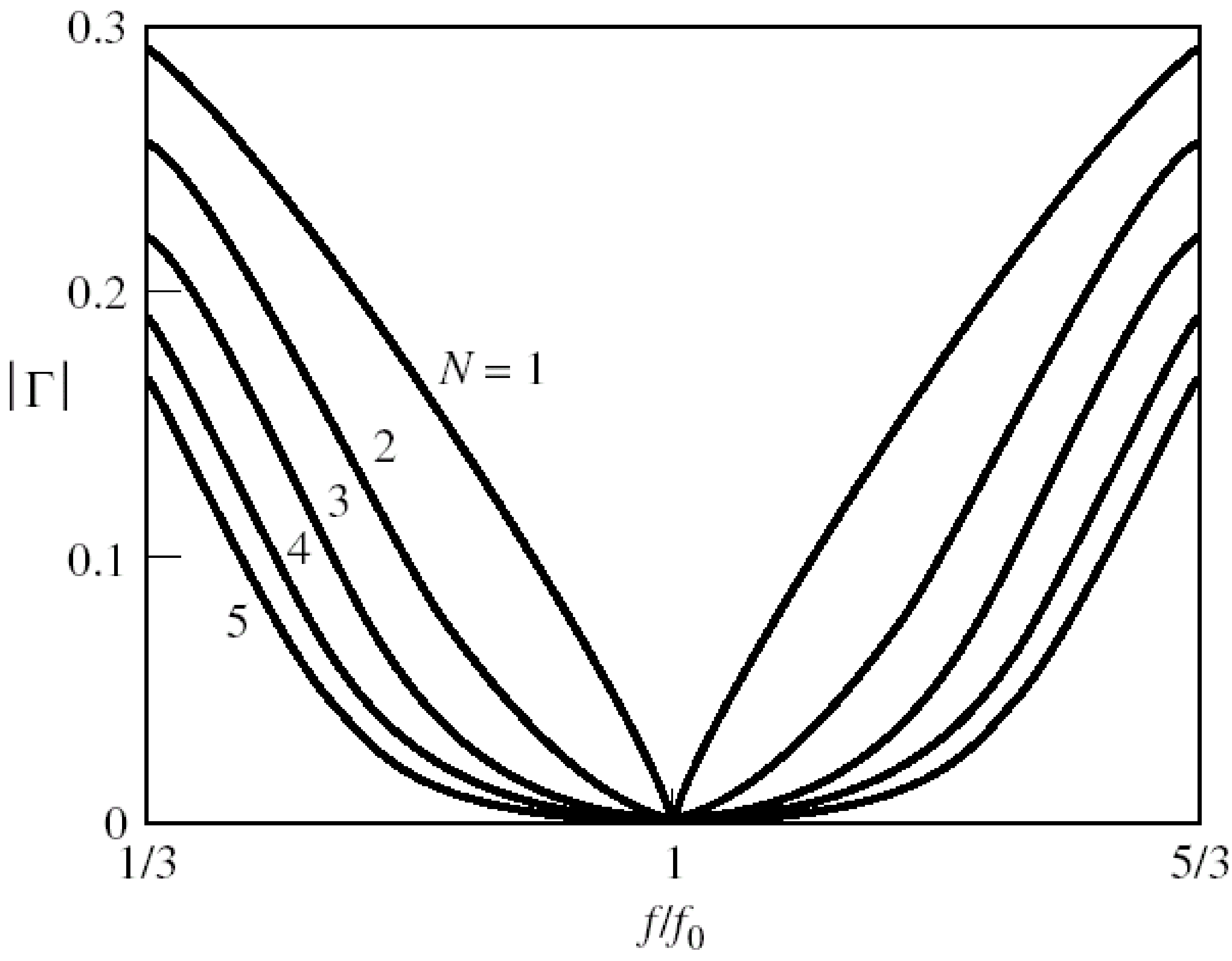
$$\Gamma_1 = \frac{Z_1 - Z_0}{Z_1 + Z_0}$$

$$\Gamma_n = \frac{Z_{n+1} - Z_n}{Z_{n+1} + Z_n}$$

$$n = \overline{1, N-1}$$

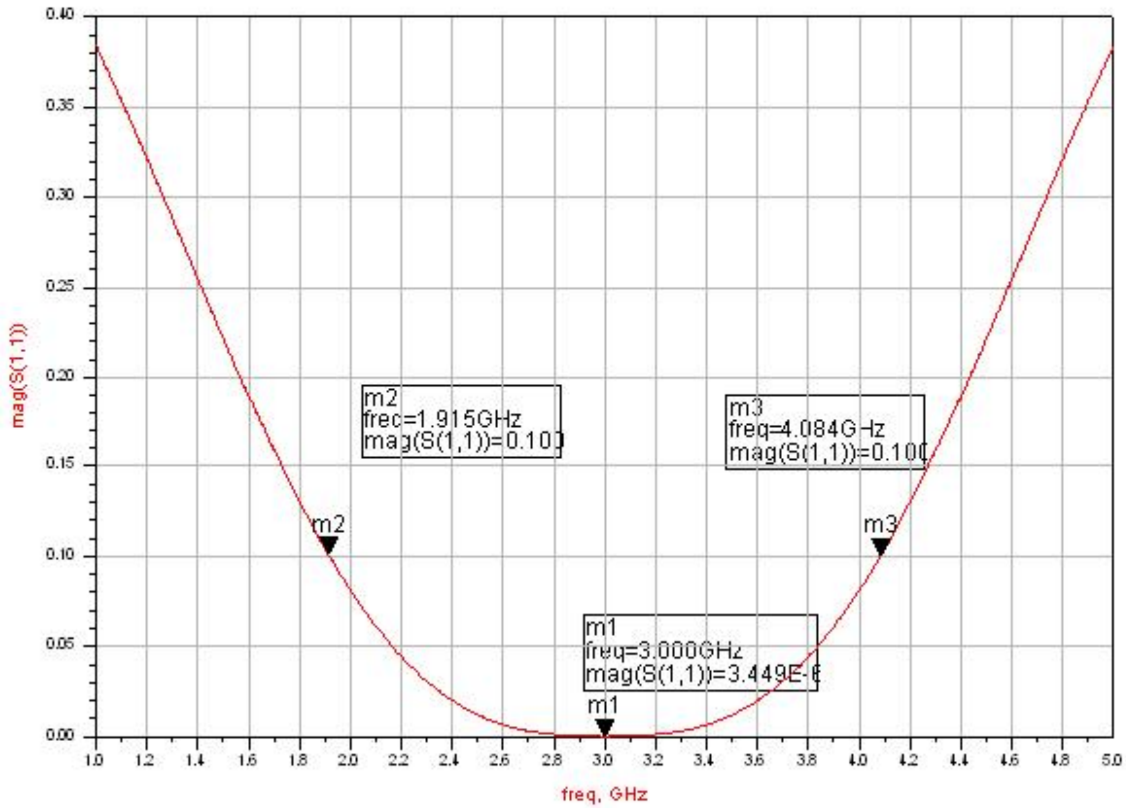
$$\Gamma_N = \frac{Z_L - Z_N}{Z_L + Z_N}$$

Bandwidth / Binomial



Simulation

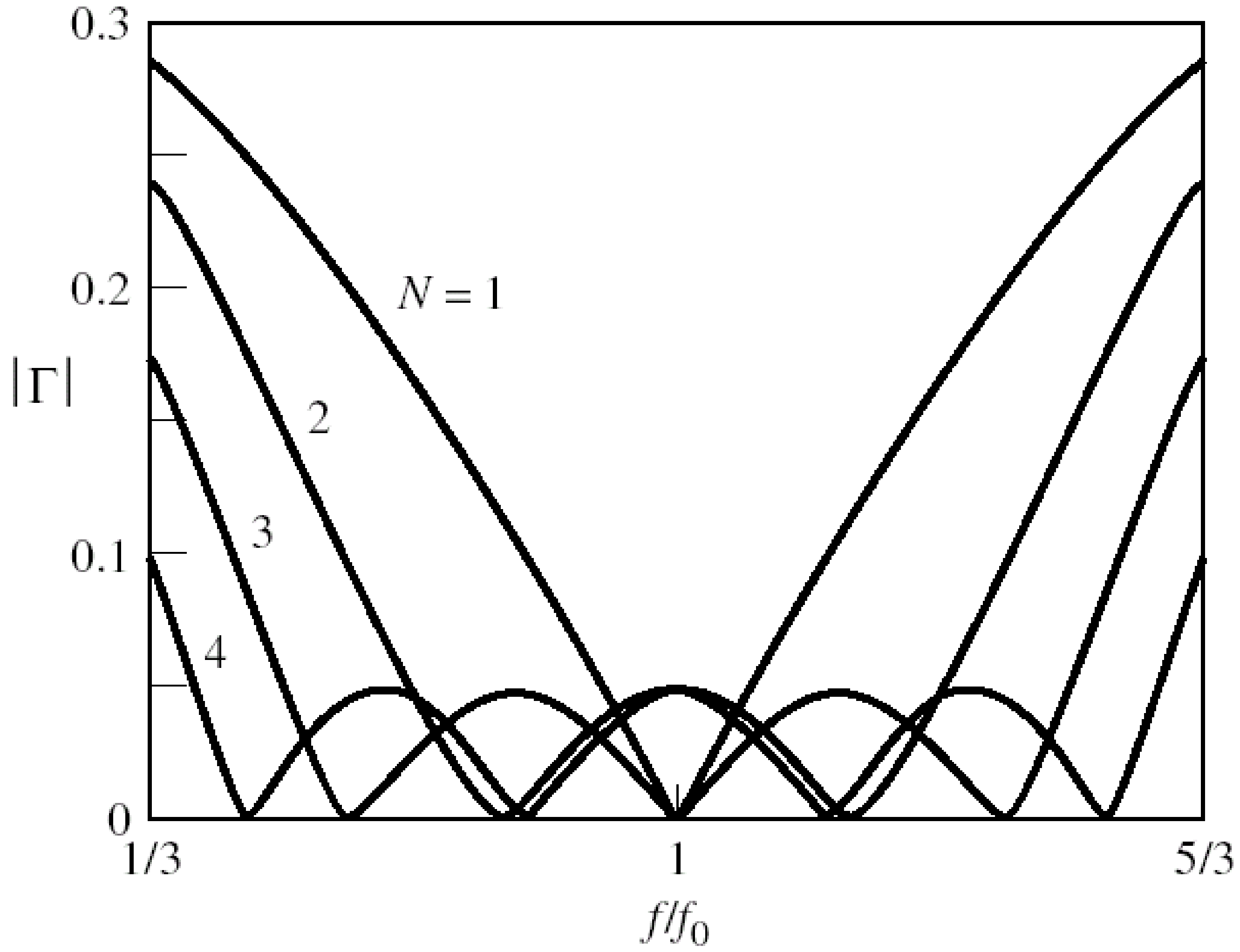
- Similarly Lab. 1



$$\Delta f = 2.169GHz$$

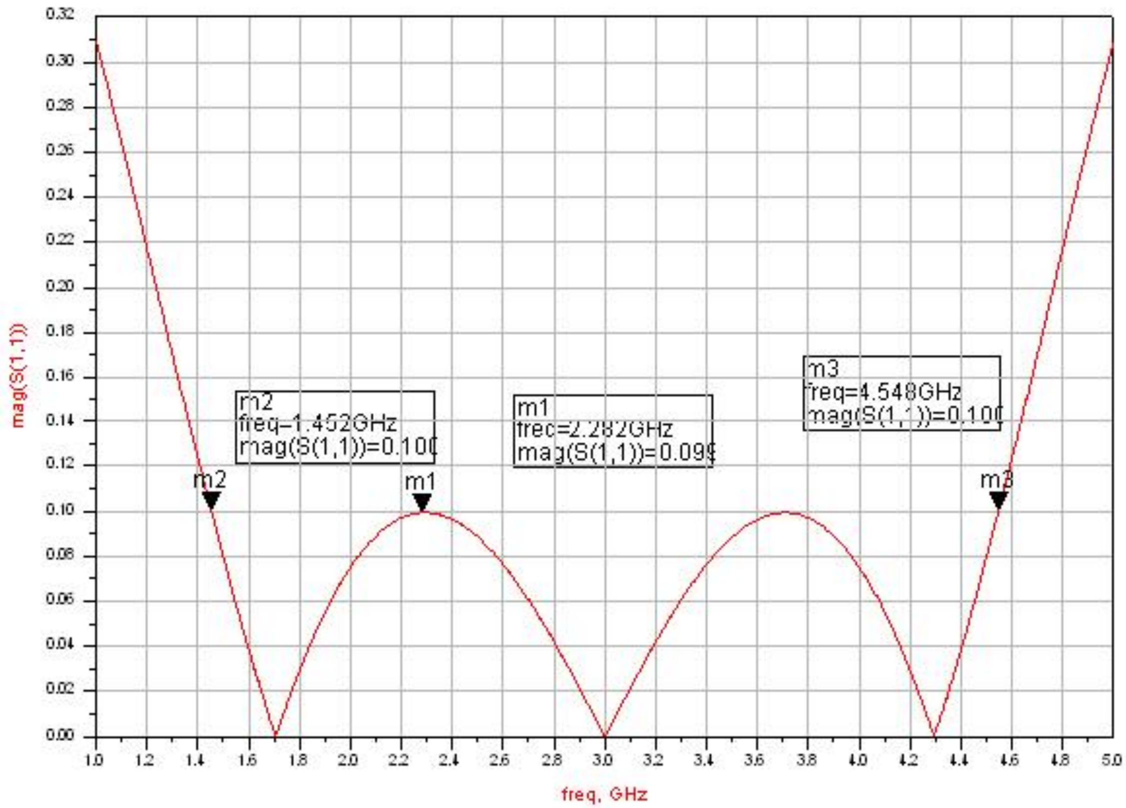
$$\left|\Gamma(3GHz)\right| = 3.5 \cdot 10^{-6}$$

Bandwidth / Chebyshev



Simultion

- Similarly Lab. 1



$$\Delta f = 3.096 GHz$$

$$\left|\Gamma\left(3GHz\right)\right| = 4.17 \cdot 10^{-5}$$

$$\left|\Gamma\left(2.282GHz\right)\right| = 0.09925$$

Contact

- Microwave and Optoelectronics Laboratory
- http://rf-opto.etti.tuiasi.ro
- rdamian@etti.tuiasi.ro